UNIVERSITÀ DI TRENTO

Growth mechanics and mechanics of nonholonomic systems

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Time	**10.30–11.30**
Room	**2R, DICAM, Mesiano**

Abstract

In this contribution, I will present the preliminary steps of an ongoing work that aims at framing the mechanics of the volumetric growth in biological media as a field theory that admits a variational formulation in spite of its being nonholonomic. The nonholonomic nature of the growth problem descends from regarding the mass balance law of a growing medium as a constraint on the time rate of a structural descriptor of the medium itself, which is compelled to equal the rate of mass variation within the body [1,2]. The structural parameter is the tensor of inelastic distortions generated by growth, and is the factor of the Bilby-Kröner-Lee decomposition of the deformation gradient tensor of the medium. The rate of mass variation, instead, is assigned phenomenologically (see, e.g., [3]), and its functional form makes it impossible, in general, to express the constraint as the time derivative of a scalar function of the generalized Lagrangian parameters of the system.

After reviewing some results on growth [1,2,4-6], I will report on a variational procedure based on the method known as Hamilton-Suslov Principle, and elaborated in [7] to handle nonholonomic constraints variationally, and I will show its connections with a more classical approach to the analytical mechanics of nonholonomic systems [8]. Then, I will discuss whether the methodology developed in [7], adapted to account for non-potential forces [9], can be applied to growth mechanics. Finally, I will summarize the main implications of this formulation, and compare it with that of other authors.

Bibliography

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