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Metamaterials with architected instabilities

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Abstract: Homogenization is used for periodic elastic grids subject to axial prestress, to obtain equivalent elastic materials. The latter can be tailored to exhibit material instabilities driven by the microstructure of the grid and its level of prestress. Instabilities include shear band fromation and Hopf bifurcation, thus leading to odd elasticity.

A design strategy is introduced for metamaterials displaying tailored instabilities. When the latter occur at the microscale, they are analyzed with a Floquet-Bloch wave technique, while analysis of macroinstabilities leads to the definition of an equivalent elastic material. This material can be obtained via homogenization theory for periodic elastic structures, subject to a state of axial prestress and incremental deformation involving axial and shear forces and bending moment [1]. Macroinstabilities in the form of shear bands usually occur only for compressive prestress, so that the stability domain for the equivalent material results unbounded in tension. We show that it is possible to design a material for which the stability domain is bounded, in other words, for which shear bands may form under tensile loads. The architecture of this structure is shown in Fig. 1 and the analysis of its dynamic behaviour leads to multiple band gaps, flat bands, and Dirac cones [2].

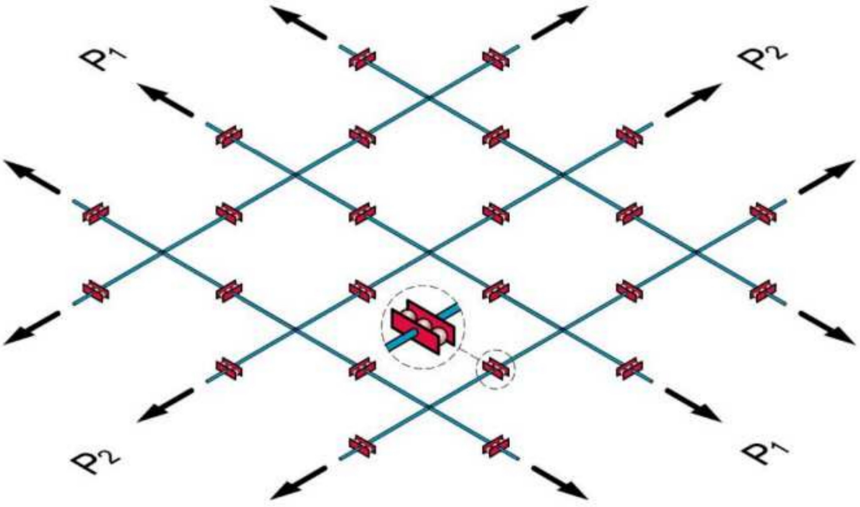

Fig. 1 - A grid of elastic and axially-preloaded rods, jointed through sliding constraints, provides an example of a metamaterial with a bounded (thus even including tension) stability domain. The fact that tensile preload may be applied precludes out-of-plane instabilities, so that the material can be used for structural membranes. In dynamics, the grid shows multiple band gaps, flat bands, and Dirac cones, while its equivalent homogeneous material provides an uncommon example of a material losing ellipticity at the parabolic boundary, thus evidencing ‘stress channelling’, corresponding to shear bands parallel (orthogonal) to the direction of the tensile (compressive) applied preload, P1 and P2.

The possibility of a Hopf bifurcation is introduced, as related to the presence of follower loads [3], or nonholonomic constraints [4], or discontinuity in the constraint curvature [5]. When this instability is implemented in a material [6], apparently work is produced in a closed strain cycle, so that conservation of energy is apparently violated. As mentioned, this violation is only apparent, as the material is able to "suck and release" energy from the environment. When a material is subject to a Hopf bifurcation, mechanical waves propagate through it without decaying, rather with amplification, because energy is extracted from the surroundings, Fig. 2.

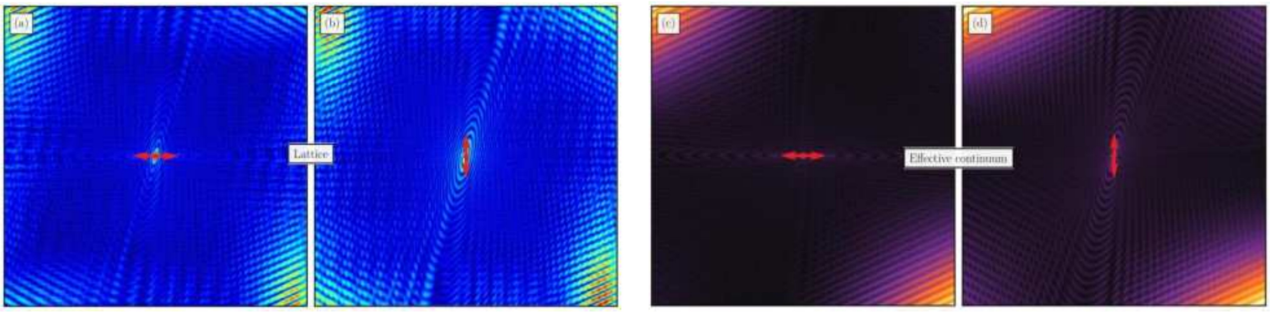

Fig. 2 - Time-harmonic forced response of an anisotropic lattice (a, b) and of the corresponding effective continuum (c, d), displaying flutter instability. The response is typical of the behavior of a dynamically unstable material, evidencing blowing-up waves traveling away from the forcing source but localized along two preferential directions.

The behaviour shown in the figure is typical of odd elasticity, violating hyperelasticity, and opens the way to new and unexpected applications in the field of metamaterials.

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