
UNIVERSITÀ DI TRENTO


# The wild complexity born from a simple nonlinearity, and how to tame it: A reprise of the "beam on an elastic foundation"

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| Room | **2R, DICAM, Mesiano** |

The problem of a beam on an elastic foundation has a long history in engineering mechanics. Its deformation and onset of buckling behavior have been employed as a surrogate for understanding an extensive list of technologically important applications, including sun-kinking of railroads, pipeline transportation engineering, and thin films on hard or soft substrates in microelectronic and biological applications. Despite this history, the global bifurcation behavior of the problem has not been fully explored. Indeed, the complexity of the equilibrium solution set will come as a surprise to many practicing and research engineers.

With an eye toward the study of creasing and deformation-localization problems in soft materials, this work studies the behavior of an inextensible infinite Euler-Bernoulli beam that is subjected to a compressive axial force and connected to a nonlinear (polynomial) elastic foundation. We seek global post-bifurcation equilibrium paths and their stability as a function of the applied force. All bifurcating paths (stable and unstable) are of interest due to the possibility of stable segments occurring on any path in the deep post-bifurcation regime of the global solution set. However, standard solution techniques (incremental Newton-Raphson coupled with strategically chosen imperfections) are foiled by the complexity of the problem's bifurcation behavior. Thus, it becomes necessary to employ sophisticated equivariant bifurcation theory and numerical methods, based on group theory, to tame the complexity and systematically obtain the desired results. These methods will be reviewed and their application to the problem at hand will be demonstrated as a typical example of their power to solve and bring order to an otherwise intractable set of equations. Finally, the results will be interpreted physically to explain the existence of localized-deformation and crease-like behavior. Parallels with phase transformation problems will also be drawn.

The seminar is organized by the Solid and Structural Mechanics Group [D. Bigoni, F. Dal Corso, L. Deseri, D. Misseroni, M.F. Pantano, A. Piccolroaz, N.Pugno, R. Springhetti]