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Title
Dynamic stability of nonlinear mechanical systems subject to non-holonomic constraints

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Abstract
The nonlinear dynamics is addressed for an elastic double pendulum subject to a non-holonomic constraint at its end. Similarly to the classical Ziegler's double pendulum subject to follower (non-conservative) loading condition, the considered system may display dynamic (flutter and divergence) instability but now even when subject to a dead (conservative) loading condition. The Method of Multiple Scales is exploited to analytically investigate the system's dynamics and determine its limit cycles. Benchmark numerical solutions corroborate the analytical findings of the investigation.

Introduction
Dynamic instabilities in elastic systems are typically associated with non-conservative follower loads [1]. Moreover, these are also connected to counterintuitive features, such as the "Ziegler's paradox", where the critical load of the system decreases in the presence of small damping compared to the undamped case. Usually, flutter and divergence instabilities, as well as the damping destabilization paradox, are considered exclusive to non-conservative loadings. However, recent studies have demonstrated that similar features can also arise in elastic systems subjected to conservative forces when non-holonomic constraints are present [2]. This work investigates the nonlinear dynamics of a double pendulum featuring a non-holonomic constraint at one end and a conservative loading device at the other (Fig. 1). Using the Method of Multiple Scales, the system's limit cycles close to Hopf's bifurcation points are determined analytically, shedding light on the dynamics induced by this configuration.

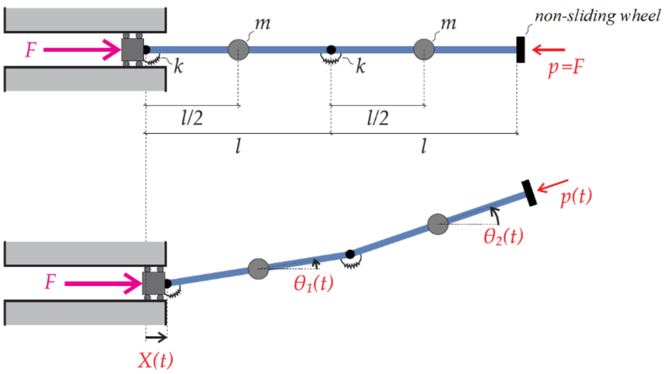

Figure 1: Reference (up) and actual (down) configurations of a double pendulum, subject at one end by a non-sliding wheel (non-holonomic constraint) and at the other by a roller and a conservative dead force *F*.

Results and Discussion
In the context of dynamic stability of elastic systems, Hopf's bifurcations can arise in visco-elastic systems subjected to conservative loads when non-holonomic constraints are present. Using the Method of Multiple Scales, it is shown that the linearized problems at different orders for the considered system share the same non-symmetric expression of the corresponding equations governing the Ziegler's double column, with the only exception regarding the force term, which now restores the system conservativity. The approximated shape of the small amplitude limit cycles has been analytically obtained and validated through comparison with numerical solutions. The developed framework paves the way for the design of innovative mechanical systems with potential applications in diverse technological fields, such as limbless locomotion.

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References
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[2] Cazzolli A., Dal Corso F., Bigoni D., Non-holonomic constraints inducing flutter instability in structures under conservative loadings, Journal of the Mechanics and Physics of Solids, 138:103919 (2020).