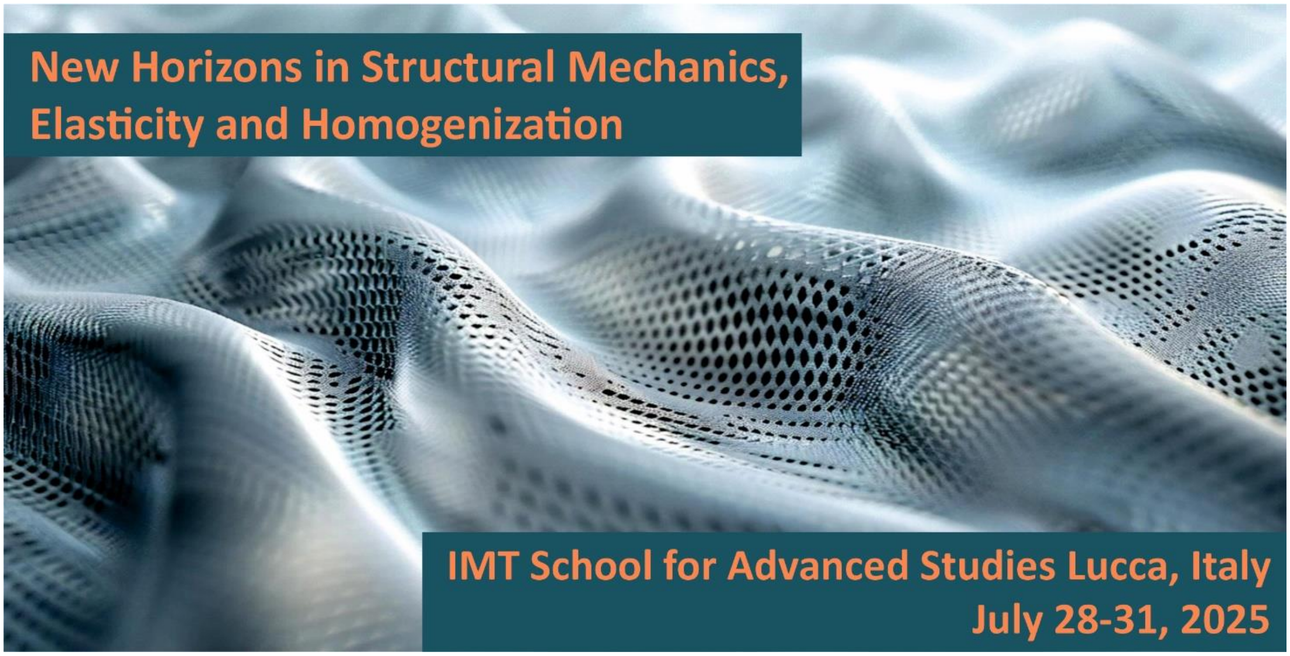
WORKSHOP PROGRAM
& BOOK OF ABSTRACTS



Organized by

Davide Bigoni – Francesco Dal Corso – Norman A. Fleck – Marco Paggi

UNIVERSITÀ DI TRENTO — IMT SCUOLA ALTI STUDI LUCCA — UNIVERSITY OF CAMBRIDGE

BEYOND

PI: Davide Bigoni

Supported by ERC-ADG-2021-101052956-BEYOND

Preface

The field of structural and solid mechanics and homogenization has witnessed exciting advancements in recent years, driven by the possibility of designing materials on the basis of ensembles of structural elements. This event, New Horizons in Structural Mechanics, Elasticity, and Homogenization, aims to explore the latest developments in these interconnected fields, shedding light on the cutting-edge techniques and applications shaping the future of engineering and design. Topics will include the role of architected and smart materials, metamaterials, metastructures, and adaptive structures, advancements in nonlinear elasticity, and multiscale homogenization techniques. The event will bring together experts to discuss new theoretical, computational and experimental methods, and the potential of advanced materials in creating more efficient and optimized structures. Participants will gain insights into how these emerging trends are revolutionizing structural systems, enabling innovations in aerospace, civil and mechanical engineering, biomechanics, and beyond. This event offers a unique opportunity to engage with pioneering work at the intersection of mechanics, materials science, and technology, setting the stage for the next generation of engineering solutions.

The event is an opportunity to celebrate the 85th birthday of Professor John R. Willis

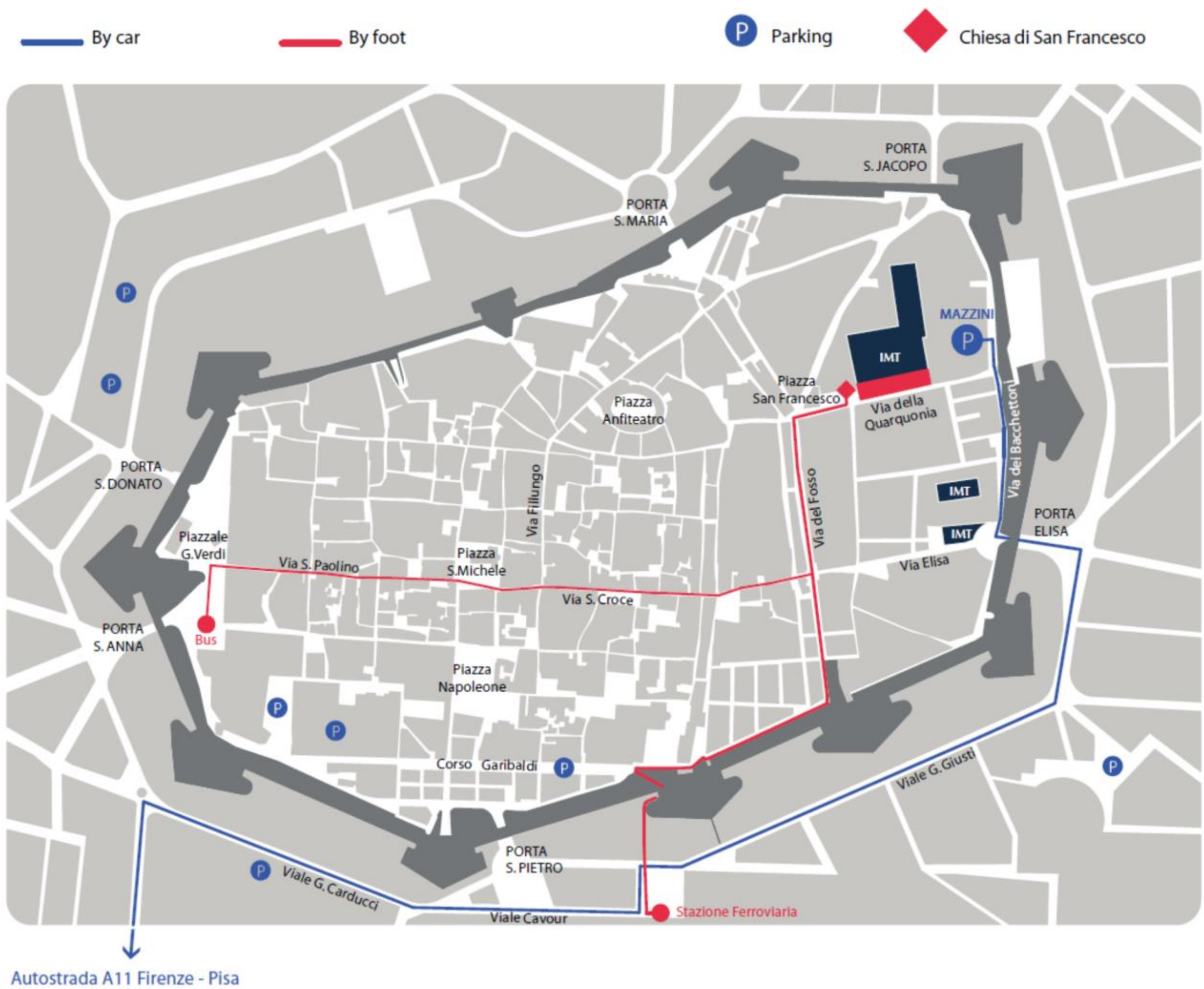
The program features 38 invited lectures delivered by internationally recognized researchers and the participation of young scholars who will present their original work during a poster session.

Lucca, July 27, 2025

Davide Bigoni, University of Trento, Italy
Francesco Dal Corso, University of Trento, Italy
Norman A. Fleck, University of Cambridge, United Kingdom
Marco Paggi, IMT School for Advanced Studies Lucca, Italy

Venue

The workshop will take place at the [IMT School for Advanced Studies Lucca](#) ([Piazza San Francesco 19](#), Lucca, Italy), from July 28 to July 31, 2025.
See the workshop website on how to reach Lucca from Pisa and Florence: https://sites.google.com/imtlucca.it/nhismeh/home



Invited Lecturers

- **Basile Audoly**, CNRS & Institut Polytechnique de Paris, France
- **Ferdinando Auricchio**, University of Pavia, Italy
- **Mattia Bacca,** University of British Columbia, Canada
- **Lorenzo Bardella**, University of Brescia, Italy
- **Katia Bertoldi**, Harvard University, USA
- **Davide Bigoni**, University of Trento, Italy
- **Francesco Dal Corso**, University of Trento, Italy
- **Luca Deseri**, University of Trento, Italy
- **Vikram Deshpande**, University of Cambridge, United Kingdom
- **Walter J. Drugan**, University of Wisconsin, USA
- **Norman A. Fleck**, University of Cambridge, United Kingdom
- **Samuel Forest**, Mines Paris PSL, France
- **Massimiliano Fraldi**, University of Naples Federico II, Italy
- **Huajian Gao**, Tsinghua University, China
- **Yonggang Huang**, Northwestern University, USA
- **John W. Hutchinson**, Harvard University, USA
- **Jean-Baptiste Leblond**, Sorbonne Université, France
- **Robert M. McMeeking**, University of California Santa Barbara, USA
- **Graeme W. Milton**, The University of Utah, USA
- **Gennady Mishuris**, Aberystwyth University, United Kingdom
- **Alexander Movchan**, University of Liverpool, United Kingdom
- **Natasha Movchan**, University of Liverpool, United Kingdom
- **Alan Needleman**, Brown University, USA
- **Sebastien Neukirch**, Sorbonne University & CNRS, Paris, France
- **Christian F. Niordson**, Denmark Technical University, Denmark
- **Giovanni Noselli**, SISSA – International School for Advanced Studies, Italy
- **Marco Paggi**, IMT School for Advanced Studies Lucca, Italy
- **Anna Pandolfi**, Politecnico di Milano, Italy
- **Sergio Pellegrino**, California Institute of Technology, USA
- **Paolo Podio-Guidugli**, Accademia Nazionale dei Lincei, Roma, Italy
- **Pedro Ponte Castañeda**, University of Pennsylvania, USA
- **Nicola M. Pugno**, University of Trento, Italy
- **Enrico Radi**, University of Modena and Reggio Emilia, Italy
- **Gianni Royer Carfagni**, University of Parma, Italy
- **Gal Shmuel**, Technion-Israel Institute of Technology, Israel
- **Valery Smyshlyaev**, University College London, United Kingdom
- **Pierre Suquet**, CNRS Centrale Marseille, France
- **John Willis**, University of Cambridge, United Kingdom

Poster exhibition

Research posters are displayed in the room where coffee breaks and lunches are served.
A snapshot on the poster will be given in Session 4.3.
The three best posters will be awarded during the scientific event closure.

Social Programme

- Welcome reception on Sunday, July 27, from 18:00;
- Cultural visit on Tuesday, July 29, from 16:30;
- Working dinner [*restricted to invited speakers only*] from 20:00.

Acknowledgements

Logistic and accommodation support from the IMT School for Advanced Studies Lucca and Fondazione Cassa di Risparmio di Lucca is gratefully acknowledged.
Support from the ERC AdG BEYOND (ERC-ADG-2021-101052956-BEYOND) is gratefully acknowledged.

Program at a glance

	Monday 28 July	Tuesday 29 July	Wednesday 30 July	Thursday 31 July	
8:30-9:00	*Registration*				
9:00-9:30	*Opening*	John Hutchinson	Alan Needleman	Pierre Suquet	9:00-9:30
9:30-10:00	John Willis	Lorenzo Bardella	Gennady Mishuris	Paolo Podio-Guidugli	9:30-10:00
10:00-10:30		Samuel Forest	Marco Paggi	Enrico Radi	10:00-10:30
10:30-11:00	*Coffee break & posters*	*Coffee break & posters*	*Coffee break & posters*	*Coffee break & posters*	10:30-11:00
11:00-11:30	Pedro Ponte Castaneda	Huajian Gao [*online*]	Gianni Royer Carfagni	Basile Audoly	11:00-11:30
11:30-12:00	Sebastien Neukirch	Christian Niordson	Gal Shmuel	Mattia Bacca	11:30-12:00
12:00-12:30	Alexander & Natasha Movchan	Francesco Dal Corso	Ferdinando Auricchio	Graeme Milton	12:00-12:30
12:30-13:30	*Lunch break & posters*	*Lunch break & posters*	*Lunch break & posters*	*Lunch break & posters*	12:30-13:30
13:30-14:00	Norman Fleck	Robert McMeeking	Jean Baptiste Leblond	*The ERC Advanced Grant BEYOND* *Promotion of diversity (subject, age, gender and geography)* *Snapshots on posters*	13:30-14:00
14:00-14:30	Nicola Pugno	Luca Deseri	Walter Drugan		14:00-14:30
14:30-15:00	Davide Bigoni	Massimiliano Fraldi	Valery Smyshlyaev		14:30-15:00
15:00-15:30	*Tea break & posters*	*Tea break & posters*	*Tea break & posters*		15:00-15:30
15:30-16:00	Anna Pandolfi	Katia Bertoldi	Giovanni Noselli		15:30-16:00
16:00-16:30	Vikram Deshpande	Sergio Pellegrino	Yonggang Huang		16:00-16:30
16:30-17:00	*Open discussion*	*Cultural visit*	*Open discussion*	Break	16:30-17:00
				Poster Award and Closure	17:00-17:30

Working dinner
(from 20:00)

Monday 28 July 2025

8:30 - 9:00 Registration

9:00 - 9:30 Opening

9:30 - 10:30 SESSION 1.1 (Chair: D. Bigoni)

9:30 - 10:30 John Willis
Beyond homogenisation

10:30 - 11:00 Coffee break & posters

11:00 - 12:30 SESSION 1.2 (Chair: M. Paggi)

11:00 - 11:30 Pedro Ponte Castañeda
Variational estimates for the macroscopic response and field statistics of elasto-viscoplastic composites

11:30 - 12:00 Florence Bertails-Descoubes, Sebastien Neukirch
A useful invariant for elastic rods

12:00 - 12:30 Alexander Movchan, Natasha Movchan
Dispersion and localisation of waves in discrete and continuous chiral elastic waveguides

12:30 - 13:30 Lunch break & posters

Monday 28 July 2025

13:30 - 15:00 SESSION 1.3 (Chair: F. Dal Corso)

13:30 - 14:00 Norman A. Fleck, John W. Hutchinson
The role of strain gradients in shear band formation at finite strain

14:00 - 14:30 Nicola M. Pugno
3D Nonlinear Avalanche Fracture Mechanics under Global Warming

14:30 - 15:00 Davide Bigoni
Homogenization for architected materials displaying strain localization, restabilization, and non-Hermitian behaviour

15:00 - 15:30 Tea break & posters

15:30 - 17:00 SESSION 1.4 (Chair: S. Forest)

15:30 - 16:00 Anna Pandolfi
A continuum-micromechanical model of the human cornea

16:00 - 16:30 I. Grega, W. Whitney, V.S. Deshpande
High-speed X-ray tomography for 4D imaging

16:30 - 17:00 Open discussion

Tuesday 29 July 2025

9:00 - 10:30 SESSION 2.1 (Chair: V. Deshpande)

9:00 - 9:30 John W. Hutchinson
Nonproportional Bending Experiments and Theory at the Micron Scale with Implications for Strain Gradient Plasticity

9:30 - 10:00 Lorenzo Bardella, Miles B. Rubin, Andrea Panteghini
Eulerian finite-deformation framework for size-dependent metal plasticity relying on elastic incompatibility

10:00 - 10:30 Mohamed Abatour, Samuel Forest
Limitations and improvements of scalar-based gradient plasticity models

10:30 - 11:00 Coffee break & posters

11:00 - 12:30 SESSION 2.2 (Chair: L. Bardella)

11:00 - 11:30 Huajian Gao [*online*]
Mechano-X: a paradigm for mechanics-centered interdisciplinary research

11:30 - 12:00 Christian F. Niordson, Brian Nyvang Legarth
On fracture of ductile periodic truss structures

12:00 - 12:30 Francesco Dal Corso
Behind the Scenes of Frictionless Sliding Sleeve Constraints

12:30 - 13:30 Lunch break & posters

Tuesday 29 July 2025

13:30 - 15:00 SESSION 2.3 (Chair: G. Noselli)

13:30 - 14:00 Robert M. McMeeking
Dendrites, voids and vacancies in solid-state lithium-ion batteries

14:00 - 14:30 Luca Deseri, Chiara Bernard, Nicola M. Pugno, Angelo R. Carotenuto, Massimiliano Fraldi
The role of mechanics during ligand-binding of transmembrane receptors across the cell membrane

14:30 - 15:00 Massimiliano Fraldi, Angelo R. Carotenuto, Arsenio Cutolo, Stefania Palumbo
Stress and natural states in growing and remodeling bio-continua

15:00 - 15:30 Tea break & posters

15:30 - 16:30 SESSION 2.4 (Chair: C. Niordson)

15:30 - 16:00 Giovanni Bordiga, Katia Bertoldi
Robotic mechanical metamaterials

16:00 - 16:30 Sergio Pellegrino
Geometrically Nonlinear Simulation of Thin-Shell Deployable Structures: Progress in 2020-2025 and Current Challenges

16:30 Cultural visit

Wednesday 30 July 2025

9:00 - 10:30 SESSION 3.1 (Chair: A.B Movchan)

9:00 - 9:30 Meng Wang, Jay Fineberg, Alan Needleman
Transition from a Crack-Type to a Spall-Type Mode of Separation for Tensile Loading of an Elastic Solid with a Weak Surface

9:30 - 10:00 Gennady Mishuris
Exploring Periodic Toughness Variations: A Case Study in Hydraulic Fracturing

10:00 - 10:30 Marco Paggi, Mohadeseh Fallah, Pietro Lenarda
Complex network materials: from statistical modelling to continuum simulations

10:30 - 11:00 Coffee break & posters

11:00 - 12:30 SESSION 3.2 (Chair: M. Fraldi)

11:00 - 11:30 Ali Haydar, Laura Galuppi, Gianni Royer Carfagni
A flux-based weak formulation of thermal problems which develops Biot's variational principle. Applications to space glazing

11:30 - 12:00 Gal Shmuel, John R. Willis
Thermal bianisotropy

12:00 - 12:30 Ferdinando Auricchio
Additive Manufacturing: design, production, modeling, computations

12:30 - 13:30 Lunch break & posters

Wednesday 30 July 2025

13:30 - 15:00 SESSION 3.3 (Chair: G. Mishuris)

13:30 - 14:00 Jean-Baptiste Leblond, Mathias Lebihain
Out-of-plane perturbation of a semi-infinite crack in a 3D infinite elastic body: a new approach to the quasistatic problem

14:00 - 14:30 Walter J. Drugan
Analytical estimates of representative volume element size via the Hill definition for random linear elastic composite materials

14:30 - 15:00 Valery Smyshlyaev
On Willis-type coupling via two-scale homogenisation of generalised elastodynamic microresonances

15:00 - 15:30 Tea break & posters

15:30 - 17:00 SESSION 3.4 (Chair: S. Pellegrino)

15:30 - 16:00 Giovanni Noselli, Ariel Surya Boiardi
Swimming by flutter instability of a hydrogel ribbon

16:00 - 16:30 Shupeng Li, Yonggang Huang
Bioelastic state recovery for haptic sensory substitution

16:30 - 17:00 Open discussion

20:00 Working dinner

Thursday 31 July 2025

9:00 - 10:30 SESSION 4.1 (Chair: J.-B. Leblond)

9:00 - 9:30 Pierre Suquet, Noël Lahellec, Renaud Masson
Internal variables arising from couplings between elasticity, viscosity and temperature in heterogeneous materials

9:30 - 10:00 Paolo Podio-Guidugli
On entropy gain and energy loss

10:00 - 10:30 Andrea Nobili, Enrico Radi, Davide Bigoni
Shear band reflection from a rigid boundary

10:30 - 11:00 Coffee break & posters

11:00 - 12:30 SESSION 4.2 (Chair: P. Suquet)

11:00 - 11:30 Manon Thbaut, Basile Audoly, Claire Lestringant
Revisiting strain gradient theory in the light of two-scale expansions

11:30 - 12:00 Mattia Bacca, Mohammad Shojaeifard
Puncture Mechanics: Snap-Through Instabilities and Friction-mediated Fracture Mechanics in Soft Solids

12:00 - 12:30 Graeme W. Milton
Guiding Stress: From Pentamodes to Cable Webs to Masonry Structures

12:30 - 13:30 Lunch break & posters

Thursday 31 July 2025

13:30 - 15:00 SESSION 4.3 (Chair: M. Paggi)
promoted by the ERC Advanced Grant
"Beyond hyperelasticity: a virgin land of extreme materials"

13:30 - 13:45 Davide Bigoni
The ERC Advanced Grant BEYOND

13:45 - 14:00 Norman A. Fleck
Promotion of diversity (subject, age, gender and geography)

14:00 - 16:30 Snapshots on posters
- Research on the ERC Advanced Grant "Beyond hyperelasticity: a virgin land of extreme materials"
- Emerging topics in mechanics

16:30 - 17:00 Break

17:00 - 17:30 Poster award and closure

Monday, July 28, 2025
9:30-10:30
Session 1.1 - Chair: Davide Bigoni

Beyond homogenisation

John Willis
Centre for Mathematical Sciences, Cambridge University
Email: jrw1005@cam.ac.uk

Keywords: Elastodynamics, random media, energy conservation

The theory of homogenisation is designed for calculating an appropriate measure of the mean disturbance, when the mean disturbance consists of waves whose wavelength is much greater than the microscopic length scale over which the properties of the medium vary. Over the last several years, the theory has been extended to be applicable to metamaterials whose effective properties include significant unusual couplings at frequencies close to the resonant frequencies of microstructural components. This presentation is concerned with the development of theory that is applicable beyond this range. For a random medium, the natural measure of the mean disturbance is the ensemble mean. This is governed by effective properties that are non-local in space and time, significantly complicating the solution of boundary value problems. A further complication is that mean waves decay with distance of propagation and yet energy is conserved. There has been so far just one configuration for which this apparent paradox is resolved. The energy lost from the mean wave is transferred during propagation to the mean-zero component of the disturbance. This was demonstrated just for the mean energy flux during time-harmonic excitation. The need for fully time-dependent solutions provides the motivation for this presentation. A new stochastic variational structure based on the principle of least action is developed, which is applicable also to nonlinear elastic response and to time-dependent microstructures. Rather than concentrating on effective properties, it permits the construction of approximations which make use of limited statistical information, applicable to each individual realisation. When the properties of the medium are time-independent, these approximations are consistent with mean energy conservation, a result stronger than that already obtained in the time-harmonic case.

Monday, July 28, 2025
11:00-12:30
Session 1.2 - Chair: Marco Paggi

Variational estimates for the macroscopic response and field statistics of elasto-viscoplastic composites

Pedro Ponte Castañeda
Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, USA
E-mail: *ponte@seas.upenn.edu*

Keywords: Nonlinear, Willis, linear comparison estimates

This work presents analytical homogenization estimates for the elasto-viscoplastic response of composites. For this purpose, use is made of incremental variational principles and of the notion of a "linear comparison composite" with Maxwell-type linear viscoelastic phases. By means of a generalization of the Willis estimates [1,2] for the macroscopic constitutive response and field statistics of the linear viscoelastic composite, corresponding estimates are obtained for the macroscopic response and field statistics of the elasto-viscoplastic composites. The final result takes the form of a set of time-dependent ODEs for the average stress tensor in the composite, as well as for the averages and fluctuation covariance of the stress in the phases. The resulting estimates provide a rigorous generalization of the variational estimates of Ponte Castañeda [3] for the macroscopic response of viscoplastic composites, reducing exactly to the latter in the fully relaxed state.

Acknowledgements

The research leading to these results was funded by the Office of Naval Research under Grant N00014-21-1-2772.

References

[1] Willis, J.R. (1977) Bounds and self-consistent estimates for the overall moduli of anisotropic composites. *Journal of the Mechanics and Physics of Solids*, **25**:185-202.
[2] Ponte Castañeda, P., Willis, J.R. (1995). The effect of spatial distribution on the effective behavior of composite materials and cracked media. *Journal of the Mechanics and Physics of Solids* **43**: 1919-1951.
[3] Ponte Castañeda, P. (1991) The effective mechanical properties of nonlinear isotropic composites. *Journal of the Mechanics and Physics of Solids* **39**: 45-71.

A useful invariant for elastic rods

Florence Bertails-Descoubes[1], Sebastien Neukirch[2]
[1]*Univ. Grenoble Alpes, Inria, CNRS, Grenoble-INP, LJK, France*
E-mail: *Florence.descoubes@inria.fr*

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Keywords: structural mechanics, dynamics, integral of motion

The static-dynamic analogy discovered by G. Kirchhoff shows that the statics of an elastic beam are equivalent to the dynamics of a spinning top. In this analogy, time and angular velocity are, for example, equivalent to arc length and curvatures. This static-dynamic analogy allows us to use Noether's theorem and unravel a quantity that is invariant along the elastic rod at equilibrium. A spinning top having a Lagrangian independent of time will have its mechanical energy constant in time. In the same manner, an elastic rod with uniform elastic properties will have the sum of its curvature energy and its axial force uniform along the structure. The invariant property is known in simple cases, see e.g. [1], but the present approach generalises it to more complex cases where extensibility, shear, gravity, and contact are involved.

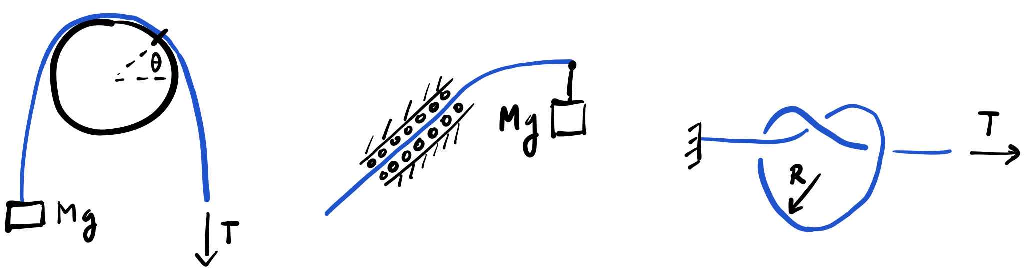

Figure 1: Three examples illustrating the use of the invariant in the statics of rods.

We will show how to compute this invariant and illustrate its usefulness in three examples. The first example is the capstan, where an elastic rod is wound around a disk. In this case, the use of the invariant easily leads to the computation of how the opening angle varies with the bending rigidity of the rod and the weight. The second example is the elastic arm scale [2], where the use of the invariant shows that the rod must have a horizontal tangent at the tip where the weight is attached. The third example is the open trefoil knot, where using the invariant leads to the relation between the applied axial force and the loop size.

Finally, we will discuss the generalisation of the present approach to the dynamics of 3D twisted rods.

References

[1] S. Kehrbaum, J. H. Maddocks. (1997) Elastic rods, rigid bodies, quaternions and the last quadrature. *Phil. Trans. Roy. Soc. A*, **355**:2117-2136.

[2] F. Bosi, D. Misseroni, F. Dal Corso, and D. Bigoni. (2014) An elastica arm scale. Proc. Roy. Soc. A, **470**:20140232.

Dispersion and localisation of waves in discrete and continuous chiral elastic waveguides

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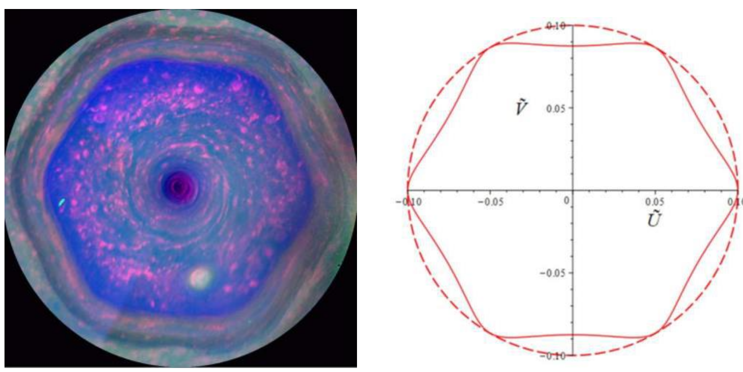
Keywords: chiral waveguides, elastic waves, wave dispersion

The lecture presents the work on modelling of dispersive elastic waves in physically chiral structures. Chirality is induced through gyroscopic forces, distributed across an elastic system, which could either be a lattice or a continuum solid. A solid is said to be geometrically chiral if it cannot be mapped to its mirror image by rigid-body rotations and translations. In contrast, a physical chirality is a feature, which reflects on a dynamic response of a gyroscopic system where the handedness of the object is determined by the direction of the gyroscopic spin (we also use the term “gyricity vector”). Illustrative examples of special “vortex waveforms” in periodic gyroscopic elastic lattice systems are discussed in the context of control of wave dispersion and localisation. Control of chiral elastic waves is also studied for waveguides, represented as periodic chains, incorporating gyroscopic spinners. Effective junction conditions, describing the chiral gyroscopic action, are used to characterise the dynamic response of the lattice. Green’s functions have been derived and analysed in different regimes of frequencies of applied forces, to describe the defect modes and wave localisation.



(a) (b)

Figure 1: (a) Polar vortex in the North Pole of Saturn (NASA’s Cassini probe 2012); (b)Trajectory of the gyropendulum approximating the polar vortex.

Illustrations and applications in understanding of polar vortex waveforms in the atmosphere, and their asymptotic approximations (see Fig. 1) are also discussed. The lecture is based on the joint work with I.S.Jones, A.Kandiah and G.Carta.

Monday, July 28, 2025
13:30-15:00
Session 1.3 - Chair: Francesco Dal Corso

The role of strain gradients in shear band formation at finite strain

Norman A. Fleck[1], John W. Hutchinson[2]
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[2] *School of Engineering and Applied Sciences, Harvard University, USA*
E-mail: *naf1@cam.ac.uk*

The formation of shear bands in a conventional elasto-plastic solid under simple shear, and in plane strain tension, is known from the literature for a wide range of material behaviours. In these classical studies, no account is taken of the role of plastic strain gradients, and it is found that J2 deformation theory gives satisfactory estimates of the bifurcation strain and post-bifurcation response. This is traced to the fact that deformation theory gives an adequate description of the material response in the total loading regime, as observed under proportional and mildly non-proportional loading. The present study considers shear band formation of an isotropic deformation theory solid at finite strain, with strain gradient effects included. The solid is taken to be incompressible, and power-law hardening. An analysis is performed both for the case of simple shear and for plane strain tension. A bifurcation analysis reveals the sensitivity of localisation strain to size of specimen in relation to material length scale. Imperfections in strength are also addressed: the growth of the band is determined as a function of imperfection magnitude and size. It is found that the width of the shear band is on the order to 20 times the material length scale associated with strain gradient plasticity.

3D Nonlinear Avalanche Fracture Mechanics under Global Warming

Nicola M. Pugno[1,2]

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Keywords: Ice/snow Avalanche**,** Fracture Mechanics, Global Warming.

In 2025, we celebrate the 85th birthday of Prof. John R. Willis, renowned for his pioneering contributions to the field of Mechanics, including Fracture Mechanics [1]. In his honor, my contribution will focus on this latter discipline [2]. This year also marks 75 years since the first ascent of an 8000-meter peak, Annapurna (Fig. 1), by Maurice Herzog and Louis Lachenal [3]. Accordingly, I have planned—hoping it will be feasible—a mountain bike crossing of Annapurna.
The celebrations of these two events are not entirely separate for me, as I wish to discuss the Fracture Mechanics of avalanches, involving both snow and ice [4] (Fig 1.). These phenomena are evolving due to global warming, posing new challenges, as highlighted by recent Italian tragedies such as the Rigopiano avalanche (snow avalanche, 18/01/17, 29 victims in a hotel) [5] and the Marmolada avalanche (ice avalanche, 03/07/22, 11 mountaineer victims) [6].
The 3D nonlinear model of avalanche Fracture Mechanics that I will present can quantify the substantial changes in avalanche triggering under climate change [6]. This research aims to help mitigate the growing associated hazards. As an example, we will examine the changes that have occurred between the birth of Prof. Willis and today, as well as those projected for the next 85 years.

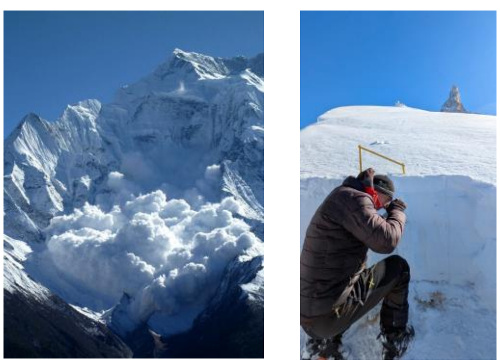

Figure 1. (left) A gigantic avalanche on the south face of Annapurna https://kathmandupost.com/miscellaneous/2016/03/27/avalanches-at-annapurna-south-continues.
(right) the author observing snow flakes in the different layers of the snowpack.

References
[1] Z. Suo, C. M. Kuo, D. M. Barnett, J. R. Willis, Fracture mechanics for piezoelectric ceramics, Journal of the Mechanics and Physics of Solids (1992) 40 (4), 739-765.
[2] N. M. Pugno, The centenary of Griffith's theory, Matter (2021), 4, 3811-3813.
[3] M. Herzog, Annapurna, The first conquest of an 8000 meter peak, 1953.
[4] N. M. Pugno, The unacknowledged risk of Himalayan avalanche triggering, Int. J. of Fracture (2014), 187, 277-283.
[5] N. M. Pugno, On the controversial role of earthquake triggering of the Rigopiano avalanche, Matter (2022), 5, 2, 372-376.
[6] C. Baroni, A. Bellin, C. Frassi, M. C. Salvatore, N. M. Pugno, L. Carturan, A. Berton, A 3D model quantifies how climate warming and ice weakening trigger mountain glacier collapses: the Marmolada case. Submitted.

Homogenization for architected materials displaying strain localization, restabilization, and non-Hermitian behaviour

Davide Bigoni

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Keywords: Metamaterials, Shear bands, Flutter instability, Hopf bifurcation.

Unusual instabilities such as tensile buckling [1], folding [2], restabilization [3], Eshelby forces [4,5], flutter [6], and self-sustained vibrations [7] have recently been identified in elastic structures.

The challenge of the ERC project Beyond is to implement these structural behaviors in architected materials to obtain equivalent solids able to display a closed domain of stability [8], multiple strain localization, restabilization (Fig. 1), and shear banding [9], and non-Hermitian behavior. The above topics will be covered in the talk.

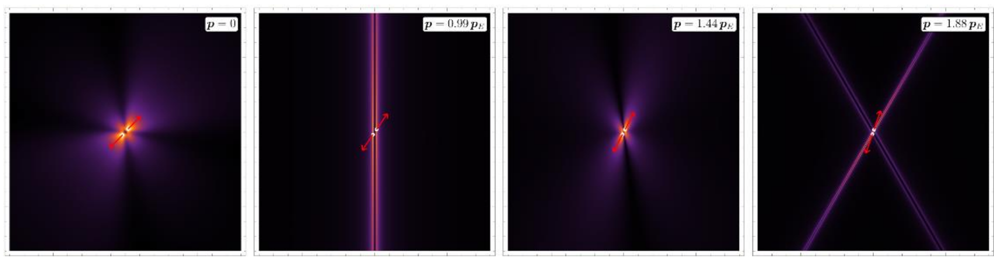

Figure 1. Displacement maps at radially increasing compressive prestress p, along a radial path emanating from an unloaded state, showing shear banding, restabilization and final shear banding.

Acknowledgements: Grant Agreement No. ERC-ADG-2021-101052956-BEYOND.

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Monday, July 28, 2025

15:30-17:00

Session 1.4 - Chair: Samuel Forest

A continuum-micromechanical model of the human cornea

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Keywords: Microstructures, Biological tissues, Computational Models**.**

The human cornea is a complex, highly specialized structure necessary for the vision function of the Eye. The cornea, due its shape and transparency, refracts and transmits the light to the retina. Cornea's mechanical properties, critical for maintaining corneal shape and function under intraocular pressure, arise from the composition of a hydrated proteoglycan-rich extracellular matrix (ECM) reinforced by an intricate network of collagen fibrils organized into lamellae. Despite extensive research, existing biomechanical models often fall short of capturing the coupled interplay between the ECM and collagen reinforcements, especially under physiological and pathological conditions. This work seeks to address this gap by proposing a novel computational model that integrates a continuum representation of the ECM with a discrete collagen-crosslinking network. The continuum approach for the ECM is chosen to represent its viscoelastic behavior and interaction with fluid flow, critical for corneal hydration and load transmission. Conversely, the collagen network is modeled as a discrete, anisotropic reinforcement system, capturing the directional stiffness imparted by the collagen fibrils and their crosslinking. The model is developed in the view to account for the influence of enzymatic degradation, age-related changes, and disease processes such as keratoconus, where alterations in the ECM-collagen coupling are known to drive structural instability.

The innovation of this approach lies in its multiscale integration, bridging the molecular mechanics of collagen crosslinking with macroscopic corneal behavior. By explicitly linking the continuum matrix with a collagen-reinforced network, the model offers some possibility to deepen our understanding of corneal mechanics. The inclusion of experimentally derived parameters for collagen alignment, crosslink density, and ECM properties, will make the model predictive in the simulation of physiological responses to intraocular pressure and external mechanical perturbations.

The work has been done in collaboration with Maria Laura De Bellis (University of Chieti-Pescara), and Christopher Miller (UK).

High-speed X-ray tomography for 4D imaging

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Keywords: High-speed imaging, architected solids, crush bands

Capturing high-rate spatiotemporal deformation of materials in three dimensions (3D) remains a significant challenge with current X-ray imaging techniques. We have developed a methodology that combines advances in neural rendering techniques with volume correlation methods to accurately reconstruct complex high-rate 3D spatiotemporal structural evolutions. The fidelity and versatility of the method, which requires no pre-training, are demonstrated for a diverse set of intricate 3D-printed micro-architected solids. Using a conventional laboratory-based X-ray tomography system we capture the elaborate 3D growth of a dynamic crush band on a timescale of less than 100 milliseconds – a more than 1000-fold speed increase of 3D reconstruction. By broadening this idea using a stereo X-ray concept, we eliminate the need to rotate the image object and thereby extend the technique to significantly faster timescales. Our neural rendering framework opens new possibilities for studying numerous poorly understood dynamic processes, such as the runaway failure of batteries and the temporal evolution of 3D shock microstructures under impact loading — all using X-ray systems available in a laboratory setting.

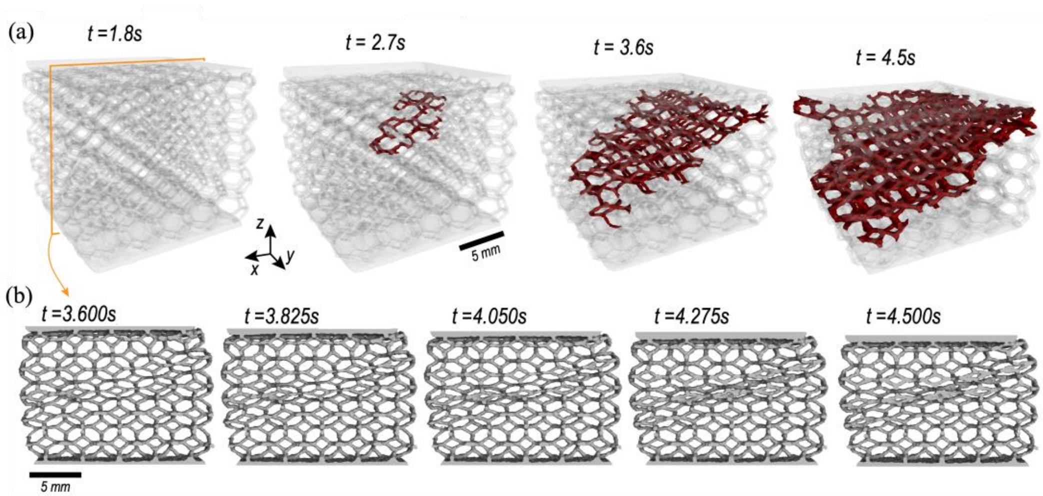

Figure 1: Millisecond time-scale 3D reconstruction of a dynamic crush band using a standard laboratory-based XCT system with a single source-detector pair. (a) Deformable NeRF 3D spatio-temporal reconstruction of the $\dot{U} \approx 1.1\ \mathrm{mm\ s^{-1}}$ uniaxial compression of the Kelvin lattice where deformation was initiated at time $t = 0$. The struts within the crush band are highlighted in red. (b) Corresponding high temporal resolution of the deformation over the final 800 ms of the deformation. The images show the vertical section indicated in (a).

Acknowledgements
The authors acknowledge funding from the UKRI Frontier Research grant award number EP/X02394X/1.

Tuesday, July 29, 2025

9:00-10:30

Session 2.1 - Chair: Vikram Deshpande

Nonproportional Bending Experiments and Theory at the Micron Scale with Implications for Strain Gradient Plasticity

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Keywords: plastic bending, micron scale, strain gradient plasticity.

This work continues the effort to develop a formulation of strain gradient plasticity (SGP) for deformation at the micron scale that is sufficiently accurate for engineering applications and yet not overly complex, The work addresses the differences between two classes of theories currently in vogue, increment and non-incremental, which arise in applications involving non-proportional plastic loading. Non-proportionality is common in metal forming processes as well as in some loading histories. The experiments have been performed by W.J. Meng and B. Zhang at Louisiana State University, while the theoretical and numerical analyses have been carried out by C. Dahlberg and T. Fischer at KTH in Stockholm, and by the speaker. The double-bend experiments are performed on Cu single crystal cantilever beams. The cantilevers loaded by end forces are first subject to bending in the vertical plane then abruptly subject to bending in the horizontal plane. In the second stage of bending, the loading device can be set to enforce either no change in the vertical displacement or no change in the vertical load. Both options are explored. Two crystal orientations have been considered. The ion-milled beams have a square cross-section with three widths tested: 2, 5 and 20 microns. An example of a bent beam is seen in Figure 1. The strength elevation in reducing the size from 20 to 2 microns is a factor of three compared to what would be expected from conventional plasticity theory. The two classes of SGP theories capture this strengthening. However, the theories differ in their prediction of the behavior in the second stage of loading due to the abrupt non-proportionality in the loading path. These differences will be highlighted and assessed with the aid of the experimental test data.

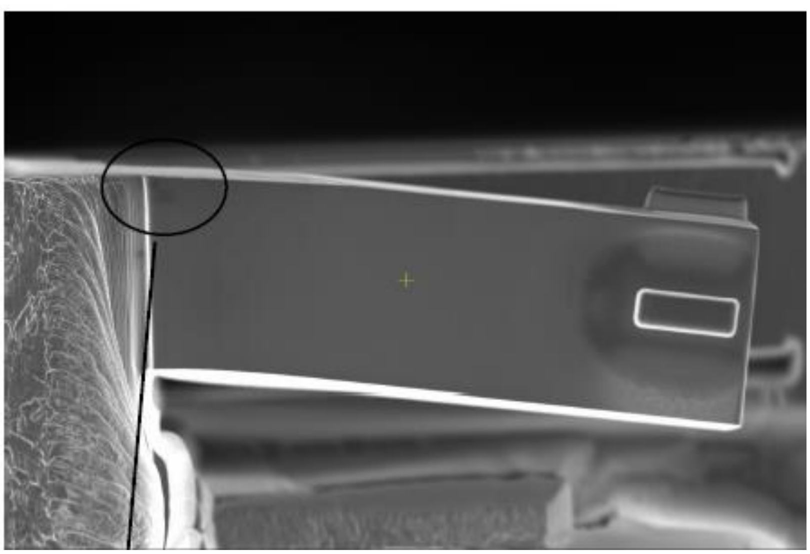

Figure 1: A micron-scale Cu single crystal cantilever beam used in the double-bend experiments.

Eulerian finite-deformation framework for size-dependent metal plasticity relying on elastic incompatibility

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Keywords: Elastic incompatibility, Eulerian formulation, Small-scale metal plasticity.

This contribution discusses two approaches, possibly to be coupled in the near future, to develop Eulerian finite-deformation size-dependent plasticity theories. The adopted Eulerian framework is free from arbitrary choices of reference and intermediate configurations as well as total and plastic deformation measures. Both approaches leverage on the incompatibility of the elastic part of the velocity gradient to introduce size-dependence [1].
By extending and modifying Gurtin's strain gradient plasticity [2], the first approach [3,4] introduces smooth transitions between elastic and plastic regimes and addresses the time-integration of the rate of the dislocation density tensor, the latter requiring, as higher-order primal variable, the use of H(curl) finite elements [5].
The second approach holds for elastically anisotropic materials, as it relies on the evolution equations for a triad of linearly independent microstructural vectors, and is a lower-order approach, thus enhancing the hardening by using gradients of the elastic strain rate [6]. Specifically, Eulerian rates of elastic incompatibilities are defined in terms of components of the negative of the current curl of the rate of plastic deformation tensor relative to distortional microstructural vectors. This definition ensures that each of these scalar rates is invariant under superposed rigid body motion and, hence, can be used independently in the Eulerian formulation of the constitutive equations. This approach is also applied to single crystals, thus unveiling the contributions to the incompatibility of both densities of geometrically necessary dislocations and crystal lattice deformation.

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Limitations and improvements of scalar-based gradient plasticity models

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Keywords: Strain gradient plasticity, Size-dependent plasticity, Strain localization.

Strain gradient plasticity models are employed to account for size effects in the plastic response of metals and polymers, on the one hand, and to simulate strain localization phenomena in the form of finite-thickness shear bands, on the other hand [1]. Scalar-based gradient plasticity models involve the gradient of a scalar internal variable, like the cumulative plastic strain in Aifantis approach. The latter introduces the laplacian of the cumulative plastic strain in the yield function, transforming the yield condition into a partial differential equation. It suffers however from two major drawbacks: The possible vanishing of the effective yield stress in the core region of an elastic-plastic beam under bending conditions, on the one hand, and the spreading of localization bands after saturation of the softening function, on the other hand. Tensor based gradient plasticity models can be used to overcome both limitations but they are computationally significantly more expensive. The use of a norm of the current plastic strain tensor instead of the cumulative plastic strain in the gradient approach will be shown to adequately represent the bending behaviour. Resorting to the gradient a saturating variable instead of the ever-increasing cumulative plastic variable allows for the simulation of finite width localization bands [2]. Such saturating variables relate to the overall dislocation densities in metals which cannot grow indefinitely. All numerical examples are provided within the micromorphic approach to gradient plasticity at finite deformations.

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Tuesday, July 29, 2025
11:00-12:30
Session 2.2 – Chair: Lorenzo Bardella

Mechano-X: a paradigm for mechanics-centered interdisciplinary research

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The classical subjects of solid mechanics, structural mechanics and mechanics of materials have played important roles in helping develop structural and functional materials, giving rise to recent advances in nanostructured materials, biomedical materials, mechanical metamaterials, soft actuators, flexible electronics, tunable mechanochromics, regenerative mechanomedicine, etc. While the classical objectives of mechanics often focus on passive access to structures and materials in existing forms, a paradigm shift, which we refer to as Mechano-X, is emerging toward proactive designing or programming of functionalities and properties of structures and materials during the manufacturing process by leveraging the fundamental mechanics principles, including our ability to quantitatively characterize the force–geometry–property/function relationships. This paradigm shift is largely driven by the rapid developments of manufacturing technologies including additive manufacturing, which have ushered in an on-going revolution in material innovations in both engineering and medicine. This technology has started to make it possible to design and construct materials and structures from nanoscale up and for the first time to fabricate materials in the laboratory that mimic biological systems in nature.

Here, we will outline the research vision behind the Tsinghua Mechano-X Institute (THUM), which has been founded as a new interdisciplinary research platform attracting scientists from mechanics, materials science, manufacturing, robotics, design, electronics, energy, sustainability, environment, biology and medicine to work together to address various challenges faced by mankind based on innovations brought forth by the on-going revolution in manufacturing capabilities. The central theme of research in THUM is to bring researchers in mechanics and various engineering disciplines together to meet, exchange, learn, collaborate, innovate, with the aim to foster the development of new sciences, new methodologies, new concepts and new ideas, as well as new technologies, new products and new markets.

On fracture of ductile periodic truss structures

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Keywords: Lattice structures, ductile fracture

A fracture mechanics theory based on the homogenized elastic properties of metamaterials is complicated by the significant impact of the intrinsic length scale of their microstructure. It has been demonstrated that a fracture theory based on the singular K-field alone is inadequate, and that the effect of T-stresses is important, even for cracks that are significantly larger than the microstructural length. Consequently, traditional standards for toughness testing fall short for these materials [1]. While the microstructure may be chosen to yield isotropic macroscopic elastic properties, the resulting fracture characteristics will exhibit anisotropy. In isotropic conventional materials it is typically assumed that a crack will grow in a direction of local symmetry, i.e. under a pure mode I crack tip load. A comprehensive fracture theory for lattice structures must address such stable directions of crack growth. In this study a periodic 2D lattice is analyzed to investigate directions of stable crack growth for different values of mode mixity and T-stress. Additionally, we quantify the role of ductility in the parent material by examining the relationship between the work of separation at the microstructural level and the applied energy release rate.

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Behind the Scenes of Frictionless Sliding Sleeve Constraints

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Keywords: Configurational mechanics; Contact mechanics; Variable-length systems

A frictionless sliding sleeve is a mechanical constraint that allows only relative sliding motion between the sleeve and the constrained element, creating an effective system with variable length. When the flexibility of the constrained component is considered, a curvature discontinuity typically arises at the sleeve exit and a tangential concentrated force, nonlinear and repulsive, is generated there.
In this presentation, I will summarize key findings on some fundamental structural systems involving this simple yet distinctive constraint, among which (i.) the self-tuning of variable-length system through transverse oscillation of the sliding sleeve constraint (Fig. 1,left) [1]; (ii.) Euler buckling through transverse compression (Fig. 1, center) [2]; and (iii.) the mechanics of flexible frictionless sliding sleeve (Fig. 1, right) [3].

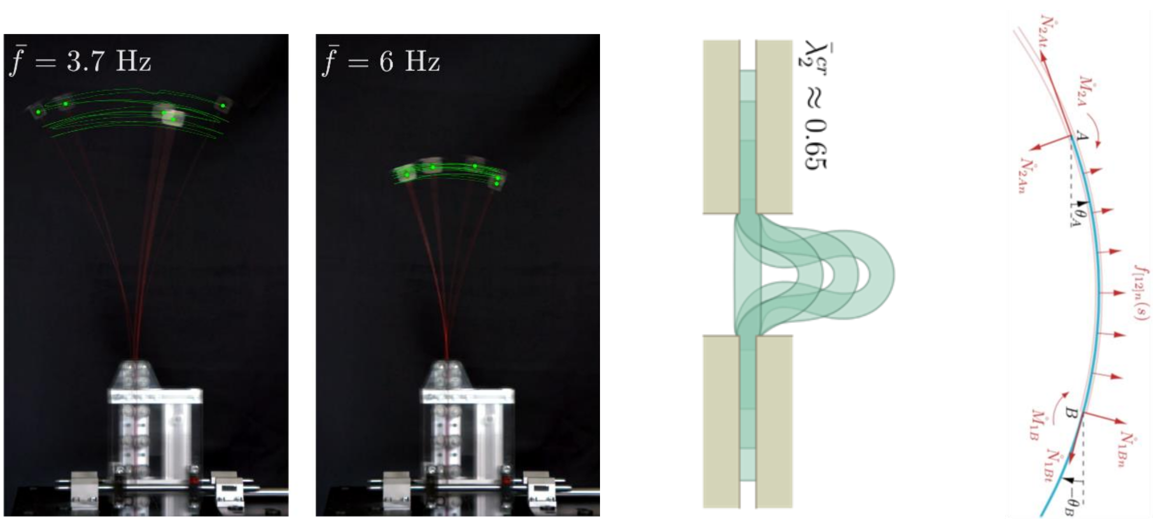

Figure 1: Self-tuning variable-length system through transverse oscillation of the sliding sleeve constraint (left); Euler buckling through transverse compression (center); and the distributed and concentrated interaction forces along a flexible frictionless sliding sleeve (right).

Acknowledgements
The author gratefully acknowledges financial support from the European Research Council (ERC) under the European Union's Horizon Europe research and innovation programme, Grant agreement No. ERC-ADG-2021-101052956-BEYOND.

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Tuesday, July 29, 2025
13:30-15:00
Session 2.3 – Chair: Giovanni Noselli

Dendrites, voids and vacancies in solid-state lithium-ion batteries

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Keywords: Lithium-ion**,** batteries, dendrites

All solid-state lithium-ion batteries consist, typically, of a lithium metal negative electrode, a ceramic solid electrolyte and a composite positive electrode consisting of cathode storage material particles in a matrix of solid electrolyte. Such batteries have superior energy and power density due to the lithium metal electrode. In addition, the design has advantages in terms of safety due to a reduced fire risk. However, such solid-state batteries are prone to lithium filaments nucleating in the electrolyte at the metal electrode. During charging of the battery, lithium invades these filaments and thickens them. This process exerts pressure on the solid electrolyte enclosing the filament, causing a lithium filled crack to propagate. This action causes a dendrite to grow across the solid electrolyte towards the positive electrode. When the dendrite reaches the positive electrode, a short-circuit occurs and the cell fails. The formation and growth of these dendrites is analyzed as a fracture mechanics problem. The treatment encompasses lithium filaments that yield plastically as well as those that remain elastic during the dendrite growth process.

Voids formed in the metal electrode have been implicated as a cause of the nucleation of lithium filaments and dendrites in the solid electrolyte. It has been claimed that vacancies in the metal electrode coalesce to form these voids due to the inability of the vacancies to diffuse fast enough to replace lithium stripped from the metal electrode during discharge of the battery. We show that there is no driving force for the diffusion of such vacancies and that void formation, if it occurs, must initiate by alternative mechanisms. Hypotheses for the nucleation and growth of voids during battery discharge are explored and models for the behavior of cavities in the metal electrode are developed during electrode stripping. The influence of interface roughness at the electrode-electrolyte junction is included in the assessment of the voiding phenomenon.

The role of mechanics during ligand-binding of transmembrane receptors across the cell membrane

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Keywords: G-Protein Coupled Receptors, Lipid rafts, Mechanobiology, Structured Deformations

This work explores the role of mechanics during ligand binding to active proteins that arise on lipid rafts across the cell membrane. These proteins, typically β-adrenergic receptors, belong to the family of G-protein-coupled receptors (GPCRs) and are embedded in the cell membrane, where they respond to specific ligands. Their signaling is mediated by the production of cyclic adenosine monophosphate (cAMP) as a result of ligand binding. This response is known to be triggered by conformational changes in the receptors and, more recently, by the simultaneous remodeling of the cell membrane. It turns out that this process is facilitated by the co-localization of lipid rafts, which provide a mechanically optimal environment for protein activation. Homeostatic levels of cAMP are regulated by multidrug-resistant proteins (MRPs), which play a crucial role in controlling this space-time-evolving process. The lipid environment surrounding GPCRs and MRPs creates a stress-confining effect, coupling the kinetics and diffusion of protein clusters with the deformation of the host membrane. Although previous research has suggested that raft formation is triggered by ligand binding, neither the experimentally observed raft coalescence nor the coupled mechanobiology and multiphysics governing these systems has been fully understood. In this talk, with a special focus on membrane-ligand-receptor complexes, we address this gap by modeling and analyzing the energetics and kinetics of the species involved. We demonstrate that the coupling of force balance, interspecies kinetics, and membrane viscosity leads to diffusion-induced membrane remodeling and raft coalescence, thereby enhancing the size of active receptor sites mediated by internal forces [1, 2].

Acknowledgements

The research leading to these results received funding primarily from the: (i) Italian Ministry of the University and Research (MUR), DICAM-EXC UNITN, Departments of Excellence 2023-2027 DM 230/2022, (ii) PRIN-2022XLBLRX, (iii) SUBBIMATT Horizon Europe EU_HE_GA 101129911, (iv) MUR 2023-2025 PNRR_CN_ICSC_Spoke 7_CUP E63C22000970007, (v) FET “Boheme” 811 86317, and (vi) ERC-ADG-2021-101052956-812 BEYOND.

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Stress and natural states in growing and remodeling bio-continua

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Keywords: nonlinear elasticity**,** cell and tissue mechanics, growth, remodeling and morphogenesis

Nonlinear elasticity is a chief actor in mechanobiology of living matter at any scale [1, 2], from highly deformable subcellular structures polymerizing under chemo-mechanical stimuli [3] to cells stretching while adhering to substrates to reach homeostatic stress for duplicating, reorienting [4] or migrating, up to the level of tissues and organs, in which geometrical compatibility, residual stresses, interstitial fluid flow and microstructural reconfigurations rule growth [5], remodeling and morphogenesis. Actually, these macroscopic processes emerge from microscopic events such as single-cell behaviors, cell-cell competition dynamics and complex multi-physics cell-extracellular interactions projected at the continuum level through a cascade of events and signal pathways across scales. This implies adopting multiscale approaches coupling mass balances, mechanical equilibrium and reaction-diffusion equations. By using Game Theory for interpreting cell logics, we discuss –with the help of selected paradigmatic examples– general and somehow counterintuitive results [6], including how stresses distribute within biological systems similarly as for plasticity, shaping the overall geometries and remodeling the elastic properties for optimizing functions, creating barriers, stopping cracks and facilitating angiogenesis and tissue regeneration. Implications for designing new classes of bio-inspired composites and metamaterials [7] are finally envisioned.

Acknowledgements

The research has received funding from the European Research Council under the European Union's Horizon Europe research and innovation programme, Grant agreement No. ERC-ADG-2021-101052956-BEYOND, the Italian Ministry of the University and Research, under the complementary actions to the National Recovery and Resilience Plan, 'Fit4MedRob —Fit for Medical Robotics' Grant (No.PNC000007) and Project AMPHYBIA(PRIN2022-2022ATZCJN).

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Tuesday, July 29, 2025
15:30-16:30
Session 2.4 – Chair: Christian Niordson

Robotic mechanical metamaterials

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Keywords: Mechanical metamaterials**,** nonlinear, sensing, computing

Flexible mechanical metamaterials are a class of structures with unique geometric features engineered to exhibit extraordinary properties in the nonlinear regime [1]. These systems have the potential to drive the next generation of smart materials and devices, enabling functionalities such as shape morphing [2], programmable nonlinear mechanical behaviors [3], and energy manipulation [4].

Recently, a groundbreaking concept has emerged: embedding computational capabilities directly into these metamaterials [5]. The integration of programmable mechanical responses, shape-shifting capabilities, and computation within a single synthetic structure paves the way for a new class of machines that are monolithic, require minimal electronic inputs, and possess advanced functionalities inherently embedded in their architecture.

In this talk, I will present our recent progress on integrating shape morphing, sensing, and intelligence into a single synthetic structure to realize such machines.

Acknowledgements

The research leading to these results has received funding from the National Science Foundation (award numbers 2041440, 2118201 and 2007278) and from the Simons Collaboration on Extreme Wave Phenomena Based on Symmetries.

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Geometrically Nonlinear Simulation of Thin-Shell Deployable Structures: Progress in 2020-2025 and Current Challenges

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Keywords: Deployable structures**,** thin shells**.**

Thin shell structures, promising high stiffness to mass ratio, efficient packaging and potentially very low cost, are leading candidates for future space missions and are standing on the front line of research in aerospace structures. A goal of this research is to develop fast simulations of folding and deployment that will enable the design optimization of large structures with complex geometries, undergoing very large, elastic deformations. Classical problems, such as the buckling and post-buckling of cylindrical shells, have been widely studied, and yet the most recent applications require geometrically non-linear simulations of unprecedented scale and complexity, including massive contact interactions, as illustrated in Fig. 1. The present contribution reviews recent advances and open problems in this field.

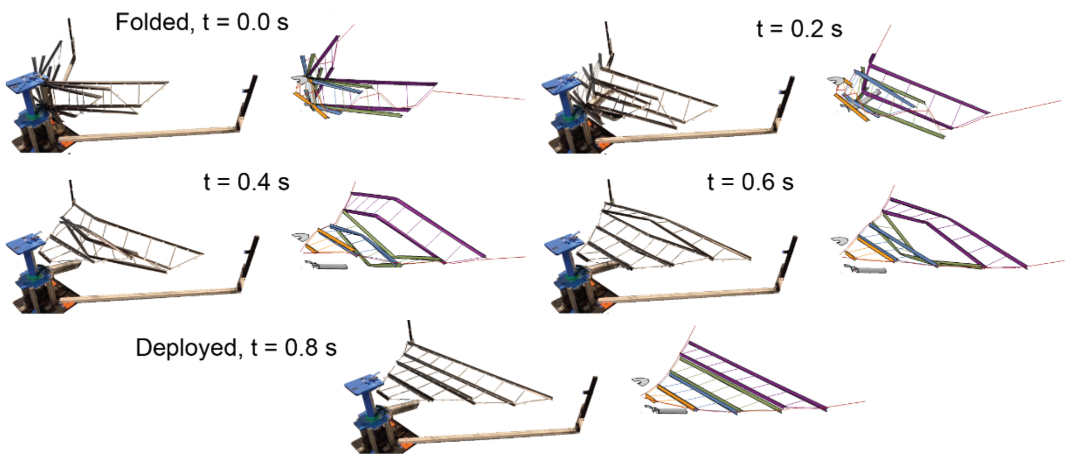

Figure 1: Snapshots comparing deployment experiment and numerical simulation for one quadrant of Caltech SSPP structure.

Acknowledgements

This research was supported by the Space Solar Power Project at Caltech.

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Wednesday, July 30, 2025

9:00-10:30

Session 3.1 – Chair: Alexander B. Movchan

Transition from a Crack-Type to a Spall-Type Mode of Separation for Tensile Loading of an Elastic Solid with a Weak Surface

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Keywords: Fracture; dynamic fracture; spall

A basic question in fracture mechanics is: how fast can a crack propagate? Classical elastic fracture mechanics predicts that, if the initial crack speed is less than the Rayleigh wave speed, the crack speed cannot ever exceed the Rayleigh wave speed, e.g. [1]. For mode II (shear) cracks, propagation at a speed that exceeds the shear wave speed occurs by nucleation of a daughter crack that begins propagation at a supershear speed and then merges with the main crack, The merged crack then propagates at a speed that is greater than the shear wave speed. This mechanism has been observed in experiments and in numerical calculations.

Mode I (tensile) cracks are seldom observed to reach the Rayleigh wave speed because micro-crack branching and/or attempted branching lead to a limiting crack speed that is less than the Rayleigh wave speed. Nevertheless, the experiments in Ref. [2] show that if branching and/or attempted branching are suppressed, mode I cracks propagating along a weak surface in an otherwise homogeneous elastic solid can propagate at a speed exceeding the Rayleigh wave speed. In addition, materials subjected to impact loading conditions can undergo spall fracture where an entire surface separates more or less simultaneously, essentially corresponding to an infinite separation speed.

Here, we report on experiments and calculations carried out to explore the transition from crack-like sub-Rayleigh wave speed to super-shear wave speed spall-like separation along a weak surface in an otherwise homogeneous elastic solid. An elastic plate with an initial edge crack subject to dynamic tensile loading is considered. The experiments are carried out on a PDMS plate comprised of two plates glued together to make the weak surface in front of the initial crack. The composition of the glue is varied to provide different cohesive properties. The calculations are carried out for an elastic solid with a cohesive surface characterizing the interface. In both the experiments and the calculations, a transition from crack-like propagation at sub-Rayleigh crack speeds to spall-like separation with an apparent separation speed that exceeds the elastic dilatational wave speed is observed. The key role of a length scale associated with the separation process is revealed by both the experiments and the computations.

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Exploring Periodic Toughness Variations: A Case Study in Hydraulic Fracturing

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Keywords: Toughness upscaling, hydraulic fracture

We discuss whether and how an averaging-based approach to material toughness can be confidently utilized. Usually, various upscaling procedures are applied to achieve the goal. Recently, we have proposed a new averaging-based approach dependent on the material and process-dependent parameters. The respective measures come from temporal averaging (in contrast to the spatial one). They require a knowledge of the instantaneous crack tip velocity during each specific process. The temporal average approach is general and can be used to analyse any stable fracture propagation process.

We used hydraulic fracture as it always produces a stable crack propagation. Numerous simulations have been performed to verify the measure proposed. We utilize an extremely accurate and effective in-house built time-space adaptive solver, which computes solutions for all classic HF models. The solver uses the crack opening and the fluid velocity as the basic unknowns in contrast to the conventional crack opening and fluid pressure pair [1]. We analyse hydraulic fracture propagating in elastic homogeneous material with periodic toughness distributions [2-3]. In particular, we show how local energy redistribution affects the process, resulting in local (in time and space) changes in the propagation regime. For example, even if the maximum and minimum values of the toughness distribution correspond solely to the high toughness regime (under a given fluid rate), a local viscosity-dominated behaviour is apparent. Another feature of the proposed measures: even though the toughness and energy release rate fracture criteria are equivalent in the problem under consideration, temporal averaging based on the energy argument appears more accurate. Finally, we show an interesting effect of the fluid reversal within the fracture for a small-time fraction and question the quasi-static approach commonly utilised in modelling propagation of HF fracture in inhomogeneous material.

Acknowledgements

The research leading to these results has received funding from the InnovateUkraine competition, funded by the UK International Development and hosted by the British Embassy in Kyiv. The author gratefully acknowledges the Industry Fellowship by the Royal Society and ERC-ADG-2021-101052956-BEYOND.

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Complex network materials: from statistical modelling to continuum simulations

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Keywords: complex materials, statistical modelling, nonlinear continuum, anisotropic materials.

Materials are inherently network-based systems, whose physical properties are defined by the structure of the chemical bonds that connect their atoms or molecules [1]. Cellulose-based materials like paper are typical examples at the engineering scale where the overall emergent mechanical response is the result of the nonlinear interplay between cellulose fibers, their progressive debonding and failure. In this lecture we present how to predict the uniaxial stress-strain response of a cross-linked random network material from a statistical micromechanics model which incorporates the key morphological features of cellulose. Then, to tackle biaxial stress states, nonlinear anisotropic elastic continua are proposed, and for the very first time a link between the two approaches is established. Several examples on in-situ testing of paper materials are provided to validate the theory [2,3]. Further insight on how to model damage inside a complex network material subject to stretching and bending is finally discussed, exploiting the new mathematical framework of phase field damage evolution on a graph [4].

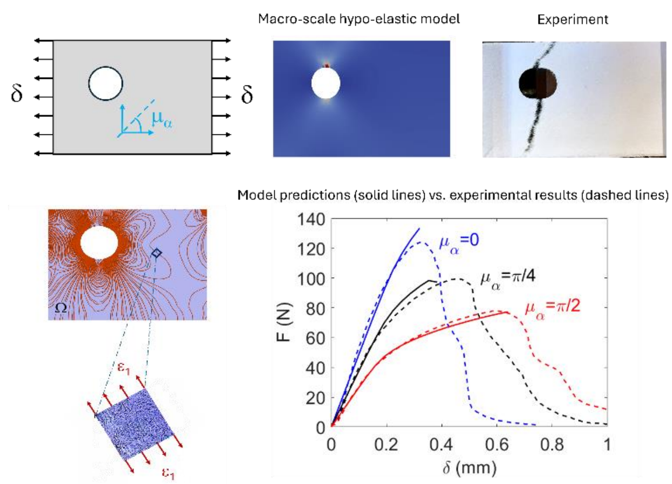

Figure 1: from statistical mechanics to nonlinear anisotropic continua.

Acknowledgements
The authors thank the IMT School for Advanced Studies Lucca, Fondazione Cassa di Risparmio di Lucca and the European Research Council under the project ERC-ADG-2021-101052956-BEYOND that made possible the organization of this event.

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Wednesday, July 30, 2025
11:00-12:30
Session 3.2 – Chair: Massimiliano Fraldi

A flux-based weak formulation of thermal problems which develops Biot's variational principle. Applications to space glazing

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Keywords: Heat conduction, variational form, finite element implementation.

In the 1950's M. Biot proposed an ingenious weak form of the heat conduction equation starting from the definition of the heat displacement vector field, whose time derivative is the heat flux. This permits a straightforward finite element implementation, but a limitation is that also the temperature field comes into play in the equations. We develop Biot's original approach in a neat formulation, where only the heat displacement appears in the variational form. Fourier's law is embedded as a holonomic constraint, while energy conservation results from the first variation (the vice-versa from Biot). Since the heat displacement is generally more regular than the temperature field, it represents a natural variable in problems with material inhomogeneities, uneven radiation, thermal shocks, interfacial (Kapitza) thermal resistances. The three-dimensional analytical set-up is implemented in a Finite Element code.

Thermal bianisotropy

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Keywords: Homogenization, Bianisotropy, Willis couplings, Metamaterials, Thermal waves, Scattering, Asymmetry

Breaking spatial symmetries can induce interactions between disparate physical fields, which manifest in the macroscopic properties of materials as cross-coupling terms. Prominent examples include Willis terms in phononics [1]; bianisotropic terms in photonics [2]; and electromomentum terms in piezoelectricity [3]. However, the development of analogous thermal cross-couplings remains limited and incomplete. Here, we develop such cross-couplings between thermodynamics and heat conduction. To this end, we introduce an exact method that is universal for describing the macroscopic dynamics of various physical processes. For heat conduction, the method shows that thermal bianisotropy emerges through the intentional design of spatial asymmetry. The method also resolves the paradox of infinite heat speed in Fourier's conduction law.

Acknowledgements

G.S. thanks Prof. Oleg Gendelman for fruitful discussions, and funding by the European Union (ERC, EXCEPTIONAL, Project No. 101045494). Funded by the European Union. Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

Funded by the European Union

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Additive Manufacturing: design, production, modeling, computations

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Keywords: Additive Manufacturing**,** topology optimization, uncertainty quantification**.**

Additive Manufacturing (AM) – also known as 3D printing – is taking off in many industrial fields, allowing for new freedom in terms of complex shapes which can be manufactured, opening the door to a new set of design possibilities (but also requirements).

After a short introduction on the technology, possible applications, current modeling challenges, the presentation will focus on the following two main aspects.

The possibility of designing and producing structures with a complex geometry puts a lot of interest toward the solution of topology optimization problems. The presentation will try to describe some attempts to frame possible formulations in a more effective mathematical setting as well as to interpret in a simple frame some interesting and very well performing algorithms.

It is well known for example that the behavior of lattice structures is not well predicted when computed on the as-designed geometry. Furthermore, due to the inherent variability of PBF-LB/M process parameters, several sources of uncertainty hinder a full understanding of the complex process-structure-property relationships; hence, a control of the source of uncertainties is extremely desired in the modeling and design process.

Wednesday, July 30, 2025

13:30-15:00

Session 3.3 – Chair: Gennady Mishuris

Out-of-plane perturbation of a semi-infinite crack in a 3D infinite elastic body: a new approach to the quasistatic problem

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Keywords: Semi-infinite crack, out-of-plane perturbation, extended Bueckner-Rice theory

The 3D quasistatic problem of out-of-plane perturbation of a semi-infinite plane crack, embedded in an infinite elastic body loaded through some arbitrary system of forces, was solved by Movchan, Gao and Willis in 1998 [1]. Their method used analytical tools specifically adapted to the infinite geometry. In contrast, the same problem is solved here using a more general approach, relying on a recent extension of the authors' of the classical Buekner-Rice theory [2]. In its original form, this theory provided the first-order expression of the variation of displacement anywhere in the body, induced by a small *tangential* perturbation of the crack front (lying within the local tangent plane); in its extended form, it provides the same result but for a general geometric perturbation of the crack front and surface, involving tangential *and normal* components. The variation of displacement is expressed as a sum of two integrals over the crack front and surface, respectively.

The extended theory is applied to Movchan, Gao and Willis's problem in three steps:

(1) Using the *extended* version of the theory and letting the point of observation of the variation of displacement go to the crack surface, we first get the variation of the displacement discontinuity across this surface, anywhere on it.
(2) We then use the *original* version of the theory to get the displacement discontinuity anywhere on the unperturbed surface, induced by certain point loads – the expression of which is required for the third step.
(3) Applying again the *extended* theory, we finally let the point of observation of the variation of the displacement discontinuity go to the crack front, so as to get the perturbed stress intensity factors there.

The results obtained fully confirm, and somewhat extend, those of Movchan, Gao and Willis [1].
Although the derivation involves non-trivial evaluations of certain limits of integrals, it reduces the treatment to this purely mathematical task, thereby circumventing the search of a method of solution of the full elasticity problem implied. This makes the method versatile and potentially applicable to other cracked geometries, closer to those of actual fracture experiments.

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Analytical estimates of representative volume element size via the Hill definition for random linear elastic composite materials

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Keywords: representative volume element, random composite material, variational principle.

The R. Hill definition of representative volume element (RVE) in composites – a volume large enough that the overall moduli are effectively independent of the (uniform) surface values of traction and displacement – is applied to the Hashin-Shtrikman-Willis [1] variational principle for random linear elastic composites. Quantitative estimates of RVE size are obtained analytically, and are compared to the analytical RVE sizes determined via a different definition of RVE introduced by Drugan and Willis [2]: the volume over which a purely local composite constitutive equation produces a small error compared to a fully nonlocal one. Since the RVE size estimates from both definitions are obtained from exactly the same variational principle and assumptions, any differences are due solely to the different RVE definitions. The approach is to employ a strain field that varies sinusoidally with position in the isotropic matrix/spherical-inclusion composites considered. One finding is that situations requiring the largest RVE sizes are a compressible matrix containing rigid inclusions, voids in an incompressible matrix, and a uniaxial strain state. Higher volume fractions of inclusions/voids are generally found to require larger RVE sizes. The results show the Hill criterion to give generally (but not always) larger – in some cases very much larger – minimum RVE size requirements than those from the Drugan and Willis definition. Example comparisons are shown in the figures below. Since a (arguably *the*) primary purpose of RVE size estimation is knowledge of when a standard purely local composite constitutive equation is accurately applicable, the Hill criterion appears to be of limited utility in such applications.

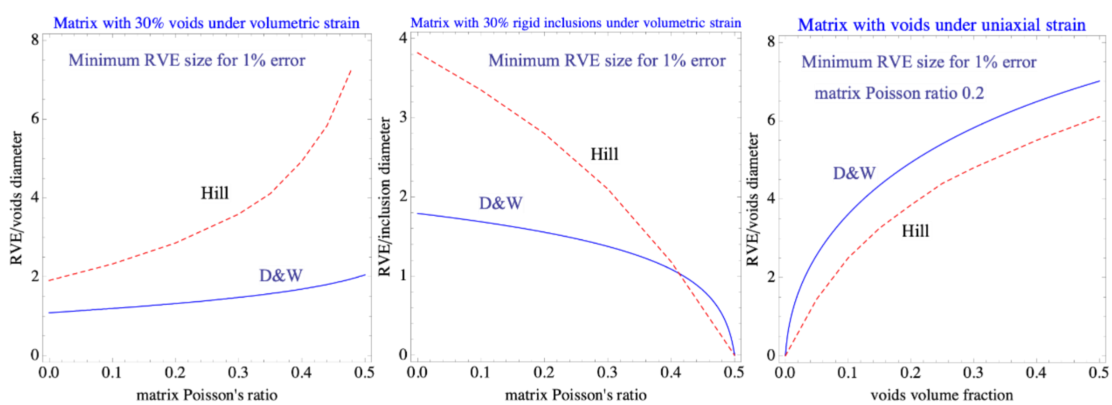

Figure 1, from left: a matrix containing 30% voids under volumetric strain; a matrix containing 30% rigid inclusions under volumetric strain; a matrix containing voids under uniaxial strain.

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On Willis-type coupling via two-scale homogenisation of generalised elastodynamic microresonances

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Keywords: Willis coupling, micro-resonances, two-scale homogenization

Back in early 1980-s John Willis, while developing his variational approximations for dynamic problems (see e.g. [1]), observed that ensemble-averaged elastodynamic constitutive relations of a composite are generally non-local and display an unconventional coupling between the mean stress and mean velocity, and mean momentum density and mean strain. Years later, this coupling effect led to an explosion of renewed interest in the context of metamaterial research, starting with work [2]: see John's (very modest) personal reflections in [3].

Such a coupling effect cannot be observed within a "classical" homogenization or a long-wavelength limit. However we argue that a "non-classical" high-contrast or micro-resonant version of homogenization, where a long wavelength of the mean field may interact with appropriate (high-contrast) subwavelength resonances (see Fig. 1 for a simplified schematics), may be capable of this. (In fact, the very word "homogenization" becomes here misleading, as the limit behavior remains intrinsically two-scale and so homogenization as classically understood fails.) In [4], resonances generally involving both interconnectedness and high anisotropy were considered, and associated coupled two-scale limit system was derived. In the present contribution, we explore how uncoupling the macroscopic part of the system in [4] may display Willis-type coupling. We discuss the issues of associated non-uniqueness, and explore a possibility of enriching the coupling effects further by allowing not only spatial but also temporal oscillations of the microscopic properties.

Control of the smallness of the errors in such (appropriately modified) two-scale approximations is a very challenging mathematical problem: see [5] for our most recent advances in this direction.

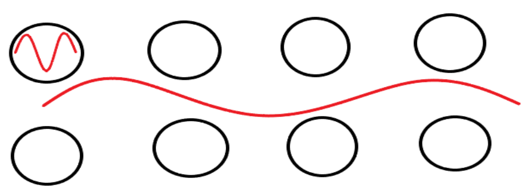

Figure 1: a basic schematics of subwavelength resonators.

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Wednesday, July 30, 2025

15:30-17:00

Session 3.4 – Chair: Sergio Pellegrino

Swimming by flutter instability of a hydrogel ribbon

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Keywords: flutter instability**,** morphoelastic filaments, soft robotics

Active filaments are ubiquitous in Nature across length scales. Examples include plant shoots and roots, which exhibit shape reconfigurations in response to external stimuli, and eukaryotic cilia and flagella, which drive micro-organisms in viscous environments by their periodic, nonreciprocal motion. Active materials can be exploited to reproduce this form of biological locomotion. However, the realization of coordinated beating patterns in artificial flagella often requires complex actuation, modulated in space and time. Drawing inspiration from the study of circumnutations in plant shoots [1], we show through experiments on polyelectrolyte hydrogel samples that directed undulatory locomotion of a soft robotic swimmer can be achieved by untethered actuation from a uniform and static electric field. We introduce a minimal mathematical model based on morphoelasticity to explain the observed behaviour. The periodic beating of the hydrogel ribbon arises from flutter instability thanks to the interplay between its active and passive reconfigurations in the viscous environment [2]. The key mechanism that underlies such behaviour is the capability of the filament to undergo active shape changes depending on its relative orientation to the external electric field, in striking similarity with gravitropism, the mechanism that drives shape changes in plants via differential growth induced by gravity. Interestingly, the hydrogel filament exhibits a form of electrotaxis whereby its swimming trajectory can be controlled by simply reorienting the electric field. Our findings trace the route for the embodiment of mechanical intelligence in soft robotic systems by the exploitation of flutter instability to achieve complex functional responses to simple stimuli. While the study is conducted on millimetre-scale samples, the design principle requires simple geometry and is hence amenable for miniaturization via micro-fabrication techniques.

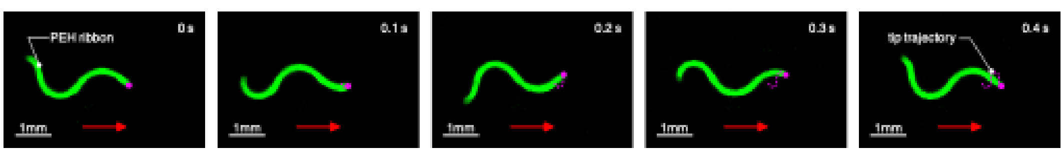

Figure 1: A polyelectrolyte hydrogel ribbon swimming in a solution of sodium chloride in water through self-sustained nonreciprocal oscillations when subjected to a constant and uniform electric field of sufficient magnitude (red arrow).

Acknowledgements

The research leading to these results has received funding from the Italian Ministry of University and Research (MUR) through the grant PRIN 2022 n. 2022NNTZNM ‘DISCOVER’ and the grant ‘Dipartimenti di Eccellenza 2023-2027 (Mathematics Area)’.

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Bioelastic state recovery for haptic sensory substitution

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The rich set of mechanoreceptors found in human skin offers a versatile engineering interface for transmitting information and eliciting perceptions, potentially serving a broad range of applications in patient care and other important industries. Targeted multisensory engagement of these afferent units, however, faces persistent challenges, especially for wearable, programmable systems that need to operate adaptively across the body. Here we present a miniaturized electromechanical structure (reference [1]) that, when combined with skin as an elastic, energy-storing element, supports bistable, self-sensing modes of deformation. Targeting specific classes of mechanoreceptors as the basis for distinct, programmed sensory responses, this haptic unit can deliver both dynamic and static stimuli, directed as either normal or shear forces. Systematic experimental and theoretical studies establish foundational principles and practical criteria for low-energy operation across natural anatomical variations in the mechanical properties of human skin. A wireless, skin-conformable haptic interface, integrating an array of these bistable transducers, serves as a high-density channel capable of rendering input from smartphone-based 3D scanning and inertial sensors. Demonstrations of this system include sensory substitution designed to improve the quality of life for patients with visual and proprioceptive impairments.

Reference

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Thursday, July 31, 2025

9:00-10:30

Session 4.1 – Chair: Jean-Baptiste Leblond

Internal variables arising from couplings between elasticity, viscosity and temperature in heterogeneous materials

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Keywords: Heterogeneous materials**,** Internal Variables, Prony series

When the deformations in heterogeneous materials are purely elastic, or purely viscous, even nonlinear, variational methods based on a single potential (either the free-energy or the dissipation potential) have been used succesfully to predict their effective behaviour (Willis [1], Ponte Castañeda and Suquet [2] among others). However, in many materials the total deformation results from the superposition of elastic, viscous or thermal strains and understanding the interplay between these different mechanisms is the principal aim of this study.

When both elastic and viscous deformations are present, or in other words when the individual constituents are governed by two potentials, it is well known that long-memory effects may result from this coupling. It is less well known that for specific microstructures these long-memory effects can be represented *rigorously* in many cases of interest by a finite (and small) number of internal variables, which can be interpreted as generalized viscous strains. The additional coupling with temperature can also be handled with internal variables, which, in most cases are also generalized viscous strains, but in specific situations can be interpreted as thermal strains associated with internal temperatures (Lahellec et al [3]).

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On entropy gain and energy loss

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Keywords: Entropy, internal energy, continuum/statistical hybridization

In continuum mechanics, two general laws rule the mathematical description of the object phenomenology, the *balance of internal energy* and the *imbalance of internal entropy*. The textbook practice is to regard both internal energy and entropy as primitive notions. At variance with this, J. Serrin showed in [1] how to consistently construct those two state functions for a constitutive class of Cauchy materials akin to compressible Navier-Stokes fluids capable to Fourier-conduct heat. He started from suitable weak forms of the postulates involving cyclic processes that are posited in the classic thermodynamics of homogeneous processes. As to entropy, he derived the Clausius-Duhem inequality, that is, the standard point-wise statement of the entropy imbalance law; he also derived an imbalance law for the internal energy.

Both imbalance laws may be written as balances, provided suitable *internal sources* are introduced, positive for entropy, negative for energy. According to Serrin, both an *internal entropy gain* and an *internal energy loss* are to be expected.

In the present contribution, building on the developments exposed in [2,3], I show how a hybridization of statistical and continuum mechanics leads to identifying, for whatever Cauchy material, the internal source of internal energy as a purely kinetic macroscopic consequence of microscopic motion randomness, a consequence that continuum mechanics alone could not possibly predict.

Acknowledgements

I acknowledge my long-lasting membership of GNFM, the Italian national group for mathematical physics.

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Shear band reflection from a rigid boundary

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Keywords: Green's function, shear bands, rigid boundary

A widespread opinion in the community of researchers working in the area of mechanics of materials is that shear bands are reflected by a rigid boundary. This opinion is based on some experimental results and a few numerical computations. However, whether this reflection may indeed be predicted theoretically, and which rules it follows remains an open question. Through the derivation of a new Green's function, valid for a prestressed elastic half-space with its straight boundary rigidly constrained, it is shown that shear bands are indeed reflected (see Fig. 1). Besides, reflection does not follow the usual Snell's law and, instead, shear bands maintain the inclination dictated by the constitutive equations of the material. This result, obtained within an incremental formulation [1] (valid for an incompressible elastic material deformed in plane strain), surprisingly leads to a closed-form solution and is illustrated with the J_2-deformation theory of plasticity, designed to mimic the behavior of metals. Our results pave the way to new applications in the realm of wave guiding metamaterials that are capable of material instabilities.

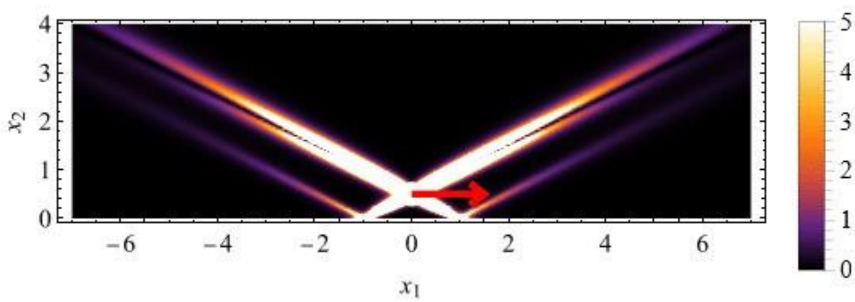

Figure 1: Strain distribution under a horizontal unit force.

Acknowledgements

The authors acknowledge financial support from the European Research Council (ERC) under the European Union's Horizon Europe research and innovation program, Grant agreement No. ERC-ADG-2021-101052956-BEYOND.

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Thursday, July 31, 2025
11:00-12:30
Session 4.2 – Chair: Pierre Suquet

Revisiting strain gradient theory in the light of two-scale expansions

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Keywords: Two-scale expansions, higher-order homogenization, strain gradient theory.

When applied to one-dimensional periodic elastic lattices, the method of two-scale expansions produces an elastic energy that depends on both the strain and the gradient of strain, and therefore fits into the 'strain gradient theory' of Mindlin [1]. The energy is expressed in terms of a macroscopic displacement u(x) and includes the scale separation parameter ε<<1 as a parameter. By design, two-scale expansions hide the microscopic features of the original problem, implying that *the minimizers u_ε(x) of the strain-gradient energy remain smooth functions of x in the limit of well separated scales, ε->0*. This key property is absent from Mindlin's theory and we show that it severely constrains the higher-order tractions that are effectively applied at the boundaries of the equivalent continuum. This special form of the boundary tractions makes it possible to rewrite the boundary energy in a compact and largely universal form. Besides, we apply a novel truncation procedure [3] to the bulk elastic energy that warrants the positivity of the homogenized elastic moduli. The *special* strain gradient model derived in this way makes it possible to revisit second-order homogenization as the minimization of a positive energy functional. The extension to dimension 2 or 3 will be briefly discussed.

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Puncture Mechanics: Snap-Through Instabilities and Friction-mediated Fracture Mechanics in Soft Solids

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Keywords: Puncture mechanics, Snap-through instability, Fracture, Nonlinear elasticity.

Puncture is a fundamental mechanical process central to biological evolution, modern medicine, and engineering. In nature, the vital success of feeding and defense often hinges on optimized puncture, enabled by the morphological evolution of claws, beaks, quills, and teeth. In technology, puncture governs processes such as material sampling, manned and automated surgical interventions, biopsy, drug delivery, and tissue injury. Despite its broad importance, puncture mechanics remain still poorly understood, primarily due to its complexity. Puncture involves (i) deep indentation [1], followed by (ii) friction- and fracture-mediated deep penetration [2–4], with the transition between (i) and (ii) governed by a snap-through instability [3]. This talk will examine key physical aspects of this phenomenon, focusing on scaling laws that capture the main mechanical determinants with predictive accuracy. Finally, outstanding challenges and open questions will be discussed.

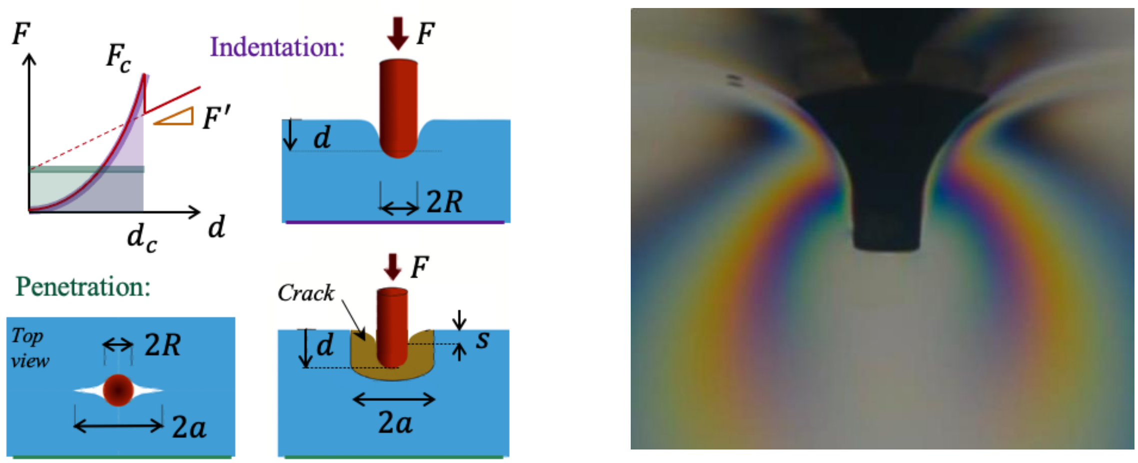

Figure 1: *Left:* Schematics of the puncture process. Indentation is energetically favored at shallow depths, and force–depth is nonlinear. Penetration involves a linear force–depth regime and is controlled by fracture and friction. *Right:* Photoelastic birefringence indicating strain distribution under a flat indenter.

Acknowledgements
The research leading to these results has received funding from the *Human Frontiers in Science Program* (RGY0073/2020), and the Natural Sciences and Engineering Research Council (NSERC) of Canada (RGPIN-2025-07085).

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Guiding Stress: From Pentamodes to Cable Webs to Masonry Structures

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Keywords: Pentamodes, Cable webs, Masonry Structures, Guiding Stress

Pentamode materials are a class of materials that are useful for guiding stress. In particular, they have been proposed for acoustic cloaking by guiding stress around objects and have been physically constructed. A key feature of pentamode materials is that each vertex in the material is the junction of four double cone elements. Thus, the tension in one element determines the tension in the other elements, and by extension uniquely determines the stress in the entire metamaterial. Here we show how this key feature can be extended to discrete wire networks, supporting forces at the terminal nodes and which may have internal nodes where no forces are applied. In usual wire or cable networks, such as in a bridge or bicycle wheel, one distributes the forces by adjusting the tension in the wires. Here our discrete networks provide an alternative way of distributing the forces through the geometry of the network. In particular, the network can be chosen so it is uniloadable, i.e. supports only one set of forces at the terminal nodes. Such uniloadable networks provide the natural generalization of pentamode materials to discrete networks. We extend such a problem to compression-only 'strut nets' subjected to fixed and reactive nodal loads. These systems provide discrete element models of masonry bodies. In particular, we solve the arch problem where one wants the strut net to avoid a given set of obstacles and also allow some of the forces to be reactive ones. This is joint work with Ada Amendola, Guy Bouchitte, Andrej Cherkaev, Antonio Fortunato, Fernando Fraternali, Ornella Mattei, and Pierre Seppecher.

Thursday, July 31, 2025
13:20-16:30
Session 4.3 – Chair: Marco Paggi
promoted by the ERC Advanced Grant
"Beyond hyperelasticity: a virgin land of extreme materials"

Snapshots on posters

Digital twin models for shoe materials and products

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Keywords: CAD-CAE integration, phase change, computational homogenization

This work focuses on the analysis of footwear materials and products through computational simulations to improve product quality and support innovative design. The integration of CAD and CAE tools is essential to streamline the design process (see Fig. 1 for real geometries). These tools facilitate efficient geometry creation, mesh generation, and simulations. By leveraging these computational models, insights can be gained to enhance footwear design, improve quality, and refine predictive capabilities for product performance throughout its lifecycle.

The study investigates critical processes of contact mechanics between the heel and ground, along with fracture and decohesion between the heel and sole in real-life usage. The analysis concentrates on stress and damage evolution due to heel-ground interactions using finite element simulations in FreeFEM++, with key scenarios including lateral impact tests, stress concentration, frictional heat generation, and fatigue-induced damage under cyclic loading.

In parallel, the computational framework includes the simulation of the injection molding process [1], particularly the solid-to-fluid phase transitions in plastics, and how thermal gradients during cooling affect residual stresses and influence the structural behaviour. This process-aware modelling incorporated into the digital twin improves predictive accuracy in both manufacturing and usage phases. Computational homogenization techniques are also employed to derive effective mechanical properties for composite materials, enabling accurate elasticity modelling in heterogeneous heel structures.

Figure 1: Finite element model of a shoe and a high heel.

Acknowledgements
The research leading to these results has received support from Tacchificio Villa Cortese, Milan, Italy and by the Italian National Recovery and Resilience Plan (PNRR), under the project "Digital Twin Technologies for Materials and Manufacturing Processes of the Footwear Industry". The authors thank support from the ERC AdG BEYOND (ERC-ADG-2021-101052956-BEYOND).

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Investigation of paper sludge properties post forming process

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Keywords: paper sludge, forming process, mechanical characterization, wettability phenomena.

Paper mill sludge (PMS) is a waste material produced in large amounts during the paper recycling industrial process. While landfilling still represents the most common *end-of-life* destination for PMS [1], the design of alternative solutions is highly relevant from a circular economy perspective. Due to its granular nature, after compaction through mechanical pressure [2], PMS may constitute an alternative eco-friendly raw material for insulation panels in buildings' interspaces and walls. In such an application context, the investigation of PMS properties is crucial to ensure minimum requirements to sustain mechanical loads as well as variations in the external environment's temperature or moisture. Controlling the panels' thickness is made also possible by exploiting natural linseed oil to perform successive compactions, since it acts as a natural binder for each PMS layer interface, favoring particles' interaction and enhancing surface adhesion [3]. Among the methods used for characterization of compacted panels, Brazilian tests were exploited to define the fracture strength and were interpreted according to the phase field approach to fracture, while low-amplitude vibration transmissibility tests and dynamic mechanical analysis were performed to highlight loss factor ηd and dynamic elastic modulus Ed, following an experimental protocol inspired by other highly porous materials [4] and considering the conditioning effect of relative humidity. Further hints on the possible development of a water diffusion-assisted fracture mechanism in PMS are pointed out to define its wettability properties.

Acknowledgements
AB, MRM, MP thank the support and funding from the Italian Government National Recovery and Resilience Plan (PNRR) and from Lucart S.p.A. DB thanks support from the ERC AdG BEYOND (ERC-ADG-2021-101052956-BEYOND).

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Instabilities and non-reciprocity in soft active structures

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Keywords: active materials, mechanical instability, non-reciprocity**.**

Soft active materials have recently drawn attention in the mechanics community as a promising platform for biomimetic actuators. For instance, such materials have been employed to reproduce flagellar beat in artificial swimmers; however, the reliance on complex actuation and controls has hindered their range of applications. Working with polyelectrolyte (PE) hydrogels as a prototypical example of artificial active materials, we investigate the complex behaviours arising from their response to electric stimuli by theoretical, numerical and experimental methods.

In [1], we found that flutter instability can be harnessed to produce self-sustained nonreciprocal oscillations in hydrogel filaments, leading to undulatory swimming under minimal actuation from a steady electric field. Building upon these findings, we explore nontrivial responses of active shells to external stimuli [2]. We adopt a nonlinear shell model of Koiter type, and we find that active shells exhibit a richer behaviour at large deformations, ensuing from the competition between stretching and bending energies. We specialize our model to cylindrical shells and elliptic plates and demonstrate the emergence of flutter through a linear stability analysis. Our analysis is confirmed by numerical simulations of the nonlinear model and by physical experiments on PE hydrogel plates. Both simulations and experiments agree in showing the achievement of oscillatory nonreciprocal motions of large amplitude.

Beyond instabilities, the responsiveness of PE hydrogels to environmental cues breaks spatial symmetry, enabling nonreciprocal transmission in both static and dynamic regimes. We focus on a periodic structure of PE hydrogel rods, exhibiting nonreciprocal behaviours, predicted by a simple mathematical model. Furthermore, the ability of PE hydrogel to harvest energy from the external stimulus allows to overcome viscous damping from the surrounding fluid and suppress attenuation, leading to unidirectional wave amplification [3].

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A parallel finite element framework for DAE-based PDEs with applications to dynamic fracture

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Keywords: High Performance Computing, Finite Element Method, Dynamic Fracture, Phase Field.

A general finite element framework is proposed for the efficient solution of systems mixing both unsteady and time-independent partial differential equations (PDEs). The combination of ODEs and algebraic equations obtained from the FEM spatial discretization leads to a differential-algebraic equations (DAEs) system. By adopting such a DAE-based approach, the framework enables the simulation of dynamic processes, including fracture evolution in solids. The implementation is developed using the deal.II finite element library [1], with support for distributed memory parallelism to ensure scalability in large-scale simulations. Time integration is handled using the IDA package of the SUNDIALS library [2], employing implicit backward differentiation formulas (BDF) of arbitrary order, with adaptive time stepping to balance accuracy and computational efficiency. The model is formulated in 3D. The flexibility and efficiency of the proposed framework makes it suitable for a broad class of DAE-based problems in computational mechanics and physics. As a demonstration, the FEM-DAE framework is applied to a dynamic phase-field model for brittle fracture, based on [3]. As well known, the model includes in fact the momentum equations for the displacements which feature second order time derivatives, and phase field equations which do not contain time derivatives. We present simulation results for two benchmark cases: a single-edge notched specimen under monotonic tensile loading, and a three-hole specimen subjected to monotonic compressive loading (see Figure 1). The results capture crack initiation and propagation accurately.

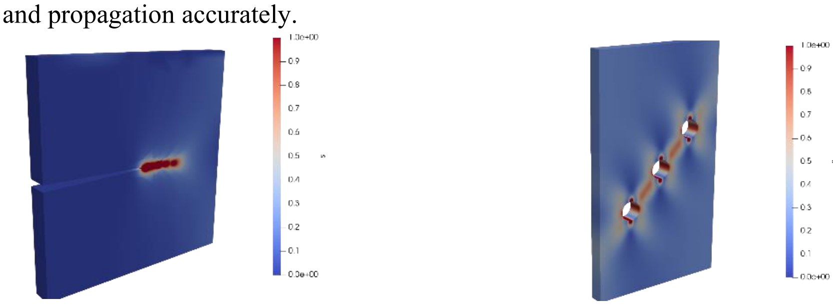

Figure 1: Results of our simulations.

Acknowledgements
The authors thank support from the ERC-ADG-2021-101052956-BEYOND.

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Poroelasticity in the presence of active fluids

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Keywords: **poroelasticity, magnetorheological fluids.**

The rehabilitation of human tissues and joints, particularly after injuries or degenerative conditions, requires innovative solutions that are both lightweight and minimally invasive. While traditional exoskeletons effectively provide support and enhance mobility, they often suffer from drawbacks such as bulkiness, rigidity, and limited adaptability to individual patient needs. As a result, there is growing interest in developing exosuits that are not only lighter and more comfortable but also capable of offering customizable support tailored to each patient's unique biomechanical requirements. Magnetorheological (MR) fluids [1], composed of micron-sized magnetic particles suspended in a carrier liquid with additives, have emerged as a promising component in adaptive materials. These active fluids behave as tunable Bingham materials, transitioning from a liquid-like to a solid-like state when exposed to an external magnetic field, resulting in a significant increase in yield stress. Their ability to dynamically adjust rheological properties in real time makes them particularly suitable for applications requiring controllable mechanical behaviour. Integrating porous deformable media with MR fluids [2] represents a promising way to achieve highly tunable mechanical devices. Porous media, in fact, are known for their lightweight properties and can provide a structural matrix whose overall mechanical response, once combined with MR fluid, can be dynamically adjusted. To characterize such a rich multi-physical system, we put forward a poroelastic formulation for porous deformable media. In the attempt to define first the response in the small-displacement regime, we begin by formulating a linearized theory. Within the framework of linear poroelasticity, thus, we build upon Biot's well-established theory of three-dimensional consolidation [3], developing both the balance of momentum and the balance of fluid content equations for the case at hand. The model accounts for the variation of viscosity of the fluid and for the viscous fluid interaction with the pore structures of the media, namely pore channel obstruction and phase transition. Thermodynamical consistency is ensured through the definition of a complementary condition that allows to switch from solid-fluid interaction to solid-solid. Lastly, validation of the formulation is obtained through several benchmarks typical of poroelasticity. The results demonstrate that the model is powerful enough to predict and optimize the behaviour of porous media containing MRFs.

Acknowledgements
The research has received funding from the European Research Council under the European Union's Horizon Europe research and innovation programme, Grant agreement No. ERC-ADG-2021-101052956-BEYOND, the Italian Ministry of the University and Research, under the complementary actions to the National Recovery and Resilience Plan, 'Fit4MedRob –Fit for Medical Robotics' Grant (No.PNC000007) and Project AMPHYBIA(PRIN2022-2022ATZCJN).

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Multiscale topology optimization via the method of moving asymptotes for architected materials

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Keywords: Method of Moving Asymptotes, Topology Optimization, Structural Mechanics, Modeling, Architected Materials

The design of architected and multiscale materials poses significant computational and methodological challenges, particularly in the context of structural topology optimization [1]. Gradient-based methods, such as the Method of Moving Asymptotes (MMA) [2], have proven highly effective in large-scale compliance minimization problems due to their robustness, scalability, and convergence properties. However, their potential for generating material architectures tailored to specific mechanical responses remains largely unexplored.
This work aims to investigate the applicability of MMA to the computational design of architected materials, with particular focus on the algorithm's integration within multiscale and homogenization-based frameworks. The study revisits the mathematical structure of MMA and explores its behavior when applied to representative topology optimization problems characterized by scale separation and nontrivial spatial organization. Emphasis is placed on understanding how algorithmic parameters, convergence behavior, and regularization strategies influence the emergence of candidate microarchitectures.
By extending a well-established optimization framework to the emerging field of materials-by-design, this research seeks to lay the groundwork for the use of MMA in the systematic synthesis of architected materials. The study is currently ongoing and constitutes a preliminary step toward bridging classical structural optimization with the computational design of advanced material systems.

Acknowledgements
Mihaela Chiappetta acknowledges support from the Italian Ministry of University and Research (MUR) through the DORIAN project ("sustainable resilient safe environment") as part of her Ph.D. program at the Department of Civil Engineering and Architecture, University of Pavia.

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Dynamics of origami-like microstructured elastic rods

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Keywords: Homogenization, Nonlinear dynamics, Origami microstructure

We study the nonlinear dynamics of a shearable elastic rod, modelled through homogenization of microstructure composed of elastic hinges and four-bar linkages, as shown in Figure 1. This microstructure enables the shear deformation and is capable of producing folding and faulting patterns [1]. In the present contribution, we extend this work to investigate their dynamical aspects. The resulting equations of motion identify an equivalent elastic rod characterized by a deformation-dependent rotational inertia. The influence of this peculiar property is investigated in terms of linear dynamics and nonlinear free and forced oscillations.

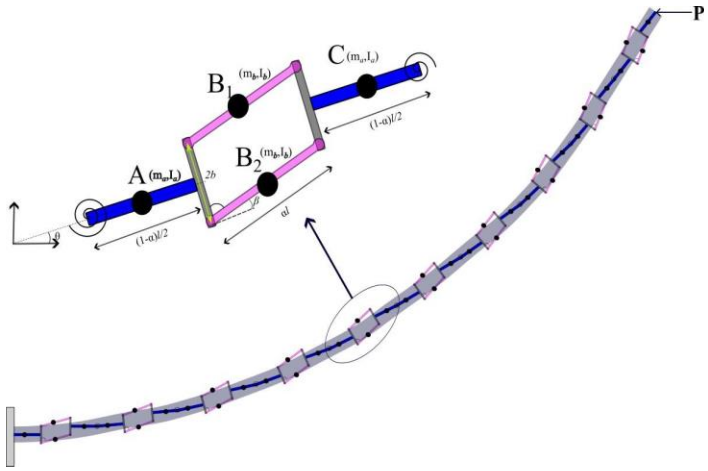

Figure 1: One-dimensional microstructured structure and equivalent shareable elastic rod. The unit-cell composed by torsional springs and four-bar linkage mechanism is highlighted.

Acknowledgements

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Modeling of the self-contact in the intestine motility

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Keywords: gastrointestinal motility, contact search algorithm, strangulation hernia.

The intestine is one of the essential parts of the digestive tract. However, our understanding of the mechanisms underlying the different types of movement, such as peristalsis and segmentation, remains incomplete, making their modelling extremely complex. Over the decades, several researchers have proposed multiphysics and multiscale models to better understand their electrophysiology, mechanics and electromechanics [1]. One of the particularities of this organ is that during its function, several parts of the intestine touch each other and can also happen during some pathological condition. To the best of our knowledge, no model proposed in the literature takes into account contact and self-contact in the modelling of intestinal contraction.

In this work, we propose an electromechanical model of the intestine, taking into account self-contact. The electromechanical model was developed using the approach proposed in [1], where we exploited the active strain approach to couple the mechanical problem to the electrophysiology problem. In addition, we proposed a contact method based on a contact search algorithm [2]. The method has been tested against a benchmark, and the results are shown in Fig 1. This allows us to study cases of self-contact as well as pathological situations like strangulation hernia.

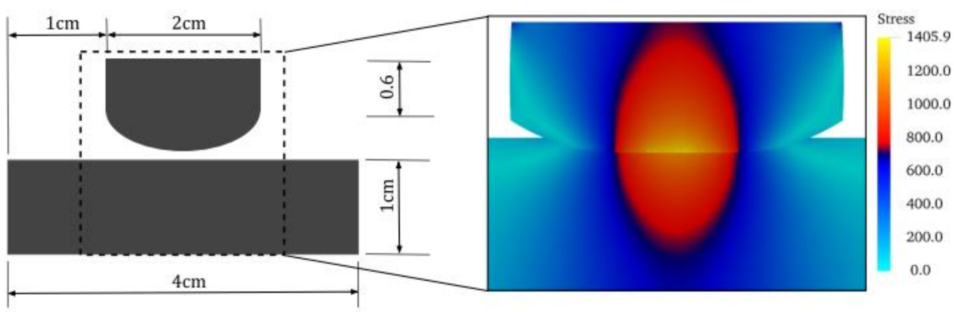

Figure 1: Benchmark problem of the compression of an elastic body against an elastic foundation.

Acknowledgements
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Wrinkling instability of a thin sheet during winding onto a rigid cylinder

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Keywords: Wrinkling, Sheet, Rotation

Wrinkles are around us, not only on the skin of humans and animals but also in industrial applications, such as wearable devices and stretchable electronics. The mechanism of wrinkling instability has been extensively studied in several boundary conditions: swelling, elastic foundations, or curved substrate. Still, the wrinkling of stretched sheets on a rigid cylinder has been less investigated so far. We perform systematic experiments for the stretched sheet on a rotating cylinder. The elastomeric sheet suspended and stretched by weight is attached to the cylinder which rotates at a constant speed. We find that the wrinkles appear and propagate along the cylinder, depending on the rotation number of rod, the sheet's aspect ratio and the loading conditions. In parallel with the rotating experiments, we consider the shape of the suspended sheet under the same loading condition, combining experiments and finite element simulations, which will help the understanding the wrinkling on the rotating cylinder. Our findings will be applied to predict and control wrinkling, which appears as an obstacle in wearable devices.

Figure 1: Model Experimental System

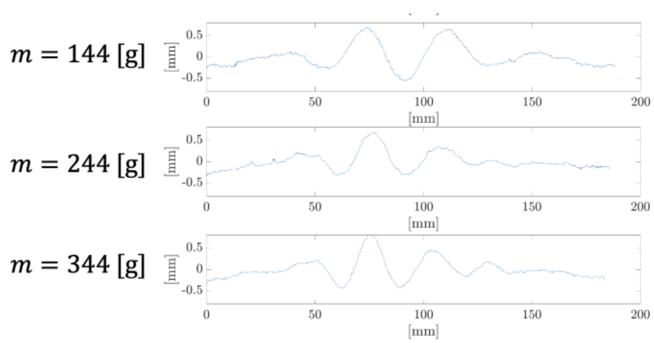

Figure 2 : The Surface of the Sheet

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Numerical simulation of deployable cable nets for the active removal of orbital debris

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Keywords: Active debris removal, deployable cable net, contact modelling

Active debris removal (ADR) will be part of any future strategy for sustainable space activities in low Earth orbits. The development of effective and adaptable capture devices is one of the main difficulties faced by ADR mission developers, together with the high costs expected. Tethered cable nets are a promising concept for the capture of derelict non-cooperative vehicles, but their modelling, analysis, and design is a non-trivial task. In fact, the net is a kinematically indeterminate, flexible structure undergoing both large displacements and finite strains. Cables have different responses to tensile and compressive forces. The tumbling target constitutes a rheonomic constraint for the net. During the capture phase, multiple unilateral contacts are expected between net and target.
We developed a finite element model for the cable net, adopting the nodal positions as the main unknowns in line with the position-based finite element formulation (PFEF) [1]. Large displacements and finite deformations are considered through the Green-Lagrange strain tensor. Cable elements are assumed to react only in tension according to a relaxed hyper-elastic constitutive law. Global damping is introduced into the model according to Rayleigh's hypothesis. Contact with the target is considered applying the method of Lagrange multipliers and by introducing slack variables to treat the inequality constraint equations. The governing equations for the nonlinear dynamical problem are solved numerically by means of the Newmark method [2].
The case of a steady obstacle of spherical shape is presented. The proposed approach turns out to be computationally effective to simulate both the deployment of the net and the capture of the debris.

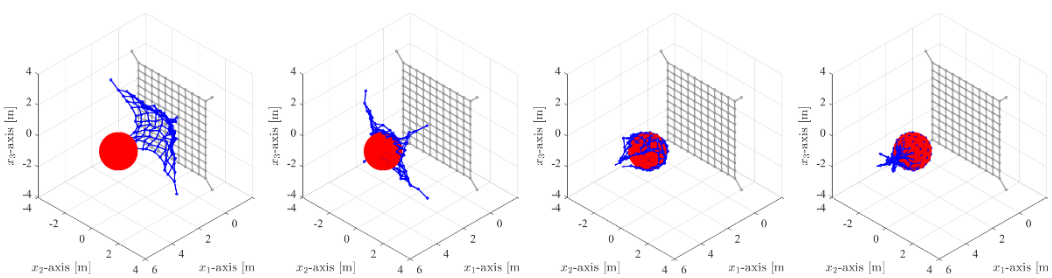

Figure 1: Simulation of the capture of a spherical debris.

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Support by the National Group of Mathematical Physics (INdAM – GNFM) is thankfully acknowledged.

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Homogenization of a grid of Timoshenko beams

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Keywords: Periodic materials; Equivalent effective continua; Timoshenko beam

Two-dimensional architected materials are typically designed as periodic grids of elastic beams. In this context, homogenization methods aim to represent periodic materials as equivalent elastic continua. Bordiga et al. [1] implemented a dynamic homogenization scheme on a lattice of prestressed Rayleigh rods, predicting the formation of shear bands in the effective medium. Viviani et al. [2] derived the elasticity tensor of the materials equivalent to elastic grids of Euler-Bernoulli beams endowed with rigid inclusions by applying a quasi-static homogenization. However, for stubby beams the Euler-Bernoulli theory yields inaccurate predictions.

In this work, we incorporate the shear deformability through the Timoshenko beam theory, which corrects the mispredictions of the Euler-Bernoulli model, and enables the determination of the elastic properties for grids composed of stubby beams --properties that are not achievable using the Euler-Bernoulli model. Figure 1 presents a proof-of-concept design that makes a square grid of stubby beams feasible. The key idea is to realize a junction that enforces full continuity between connected beams, operating on different planes while preserving the out-of-plane symmetry. This type of junction allows the beams to be extremely stubby, preventing material interpenetration. Our findings show that the use of stubby beams leads to mechanical properties of the equivalent material that are unachievable with slender beams.

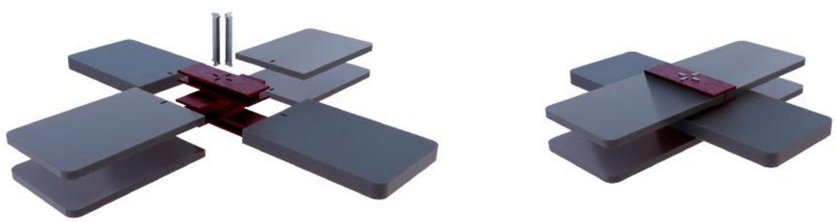

Figure 1: Concept model of a multi-layer rigid joint for a square geometry. An exploded view (left), and a perspective view (right).

Acknowledgements

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Bifurcation of a coated disc under different, but uniform, radial forces

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Keywords: Bifurcation, Complex potentials, Elastic rod theory

Cylindrical structures such as rings and arches exhibit instabilities under external radial force distributions, typically evidencing an oval-shaped bifurcation mode. This behaviour is significantly altered by the presence of an internal material. An analytical solution is derived using a complex potential formulation [1] to describe the bifurcation behaviour of an elastic disc coated with a circular elastic rod [2]. Both perfect and incomplete bonds at the disc-coating interface are considered, and the effects of distinct types of radial loads are captured [3]. The mechanical properties and loading conditions of the coating–disc system, determine a broad spectrum of bifurcation modes, ranging from low-order ovalization to high-order undulatory patterns (Figure 1). The theoretical framework and results are relevant to a wide range of applications, including coated fibre systems, and biologically inspired phenomena such as plant and fruit morphogenesis.

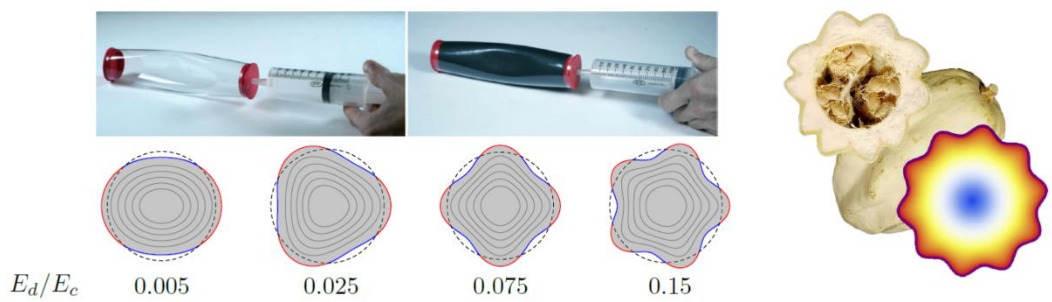

Figure 1 - Upper part: bifurcation of a tube under pressure and of the skin of a pumpkin during maturation. Lower part: bifurcation modes for different values of the ratio between the Young modulus of the disc (E_d) and of the coating (E_c).

Acknowledgements
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On a mass flux-preserving hybrid element for finite-strain poroelasticity

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Keywords: Hybrid Finite Element, Poroelasticity, Variational Principles

The computational modeling of coupled diffusion in the finite strain regime poses significant challenges due to the nonlinear interplay between solid deformation and fluid transport within an interconnected pore network.
Standard two-field finite element formulations, based on solid displacement and pore pressure [1], lead to saddle point variational problems requiring stabilization procedures to satisfy the discrete inf-sup condition [2] and suppress spurious pressure oscillations. While effective, such techniques or mesh refinements increase computational cost and can significantly affect the accuracy of the solution even for seemingly simple benchmark problems.

An alternative class of formulations [3,4] involving a different set of state variables allows for a variational minimization principle that inherently satisfies the inf-sup condition *a priori*. These methods offer an element with superior intrinsic stability but entail additional degrees of freedom and increased implementation complexity in the finite deformation setting, particularly due to the need for Piola transformations and the careful treatment of geometric mappings. Moreover, specialized numerical integration for the volumetric coupling term is required.
Motivated by recent developments in the small strain regime [5], the present work presents a novel hybrid flux-preserving finite element formulation, designed to preserve mass flux consistency within each element, by adopting an alternative set of primary variables.
The solid deformation and the mass flux fields are adopted as primary unknowns, while the fluid potential acts as a Lagrange multiplier to enforce weak continuity of mass flow across interelement boundaries, thus avoiding the necessity of globally conforming function space.
The element formulation is introduced and implemented in an open Finite Element Program (FEAP). Its performance is examined on classical benchmark problems for poroelasticity, with pressure accuracy being evaluated as a main feature of the proposed approach. The method ensures local mass balance, avoids globally conforming spaces, and removes the need for external stabilization, providing a stable and accurate framework for large strain poromechanics.

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Geometrically nonlinear analysis of planar Kirchhoff rods through a position-based finite element formulation

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Keywords: position-based finite element formulation**,** thin Kirchhoff rod, Hermite interpolation

This contribution addresses the nonlinear static analysis of planar Kirchhoff rods, accounting for large displacements and finite strains. To this aim, the position-based finite element formulation developed by Valvo [1] for isoparametric elements is extended to include rotational degrees of freedom. Accordingly, the nodal positions and rotations in the current configuration are chosen as the main unknowns. A curved planar Kirchhoff rod element is formulated adopting an interpolation scheme based on Hermite polynomials, as first proposed by Armero [2]. Our formulation stands out for the following advantages: it is a positional model that includes rotational degrees of freedom; it can be used for both finite and infinitesimal displacements, in the latter case after a consistent linearisation; it furnishes simple analytical expressions of the secant and tangent stiffness matrices; the secant stiffness matrix turns out to be symmetric; the model can be applied to both straight and curved rods. It is also noteworthy that any hyper-elastic constitutive law can be easily implemented. In this work, for the sake of illustration, the constitutive law derived from classical laminated beam theory is adopted, potentially allowing for elastic couplings to be considered. The governing equations are solved using an incremental-iterative procedure based on Newton-Raphson's method. The formulation is effective in both linear and nonlinear cases, as shown by the excellent agreement with both analytical solutions and Abaqus® results.

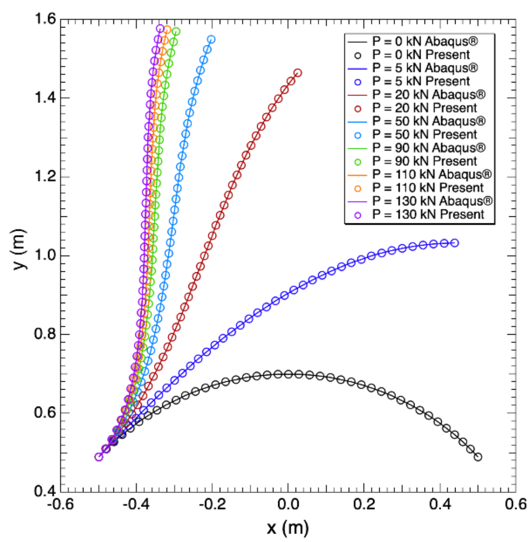

Figure 1: Cantilever circular beam under a transversal dead load at its tip: reference and deformed configurations at different load levels.

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Periodic beading in soft cylinders: the role of surface elasticity

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Keywords: Plateau—Rayleigh instability, Elastocapillarity, Buckling, Surface elasticity, Hyperelasticity

The Plateau-Rayleigh instability shows that a cylindrical fluid flow can be destabilized by surface tension. Similarly, capillary forces can make an elastic cylinder unstable when the elastocapillary length is comparable to the cylinder's radius.

While existing models predict a single isolated bulge as the result of an instability, experiments reveal a periodic sequence of bulges spaced out by thinned regions, a phenomenon known as beading instability. Most models hypothesize that surface tension is independent of the deformation of the solid, neglecting variations due to surface stretch.

Here, we assume that surface tension arises from the deformation of material particles near the free surface, treating it as a pre-stretched elastic surface surrounding the body. Using the theoretical framework proposed by Gurtin and Murdoch, we show that a cylindrical solid can undergo a mechanical instability with a finite critical wavelength if the body is sufficiently soft or axially stretched. Post-buckling numerical simulations reveal a morphology in qualitative agreement with experimental observations. Period-halving secondary bifurcations are also observed. The results of this research have broad implications for soft materials, biomechanics, and microfabrication applications where surface tension plays a crucial role.

Acknowledgements

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Exploiting instabilities for innovative energy absorber metamaterials

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Keywords: metamaterials, snap-through, energy absorption.

In recent years, research has focused on the study of lightweight metamaterials capable of absorbing large amounts of energy. The idea of combining low weight with the energy-absorbing capability can be achieved by exploiting mechanical instability. This concept allows the modulation of the first critical mode (micro- or macro-scale energy dissipation [1-3]) and the tailoring of the stress-strain curve of metamaterials [4]. In this way, instability is no longer a weakness in the structure, but a potential resource that can be exploited to realize programmable materials. In this work we show how, by appropriately studying the micro-geometry of the metamaterial, it is possible to tailor the first critical mode and the stress-strain curve by exploiting the snap-through instability. In particular, an innovative 3D truss-metamaterial is analyzed. It is composed of a unit cell that can be tessellated in space. The equilibrium path of the single cell is found by imposing suitable periodic boundary conditions and multi-critical points can be observed, related to sequential snap-through instabilities. By varying the geometrical parameters of the unit cell, the mechanical response and energy absorption can be tailored. The agreement between the analytical results and the FEM results is remarkable. Moreover, a non-linear optimization problem is carried out to maximize the ratio between the energy absorbed and the weight. We argue that this metamaterial can be used as a lightweight energy absorber able to dissipate a large amount of energy.

Acknowledgements

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High-fidelity computational framework for rough contact problems

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Keywords: Contact mechanics, roughness, fracture, wear

Accurate modeling of rough contact problems is central to computational mechanics, as surface topography strongly influences stress transfer, friction, adhesion, wear, and fracture. This work presents a robust and flexible simulation framework based on the so-called eMbedded Profile for Joint Roughness (MPJR) interface finite elements [1], which enables high-fidelity rough contact modeling. The key idea consists of treating the contact interface as nominally smooth while the actual surface height field is embedded directly into the interface finite element, avoiding the computationally costly discretization of the surface. The MPJR interface finite element methodology has been successfully applied to frictional and adhesive contact problems [2,3], integrating experimental surface data acquired through Atomic Force Microscope (AFM) (Fig. 1). Moreover, the MPJR interface finite element has been coupled with a phase-field model for brittle fracture in [4] to simulate contact-induced cracking under smooth and rough spherical indenters, demonstrating that the method can effectively solve highly nonlinear contact problems.

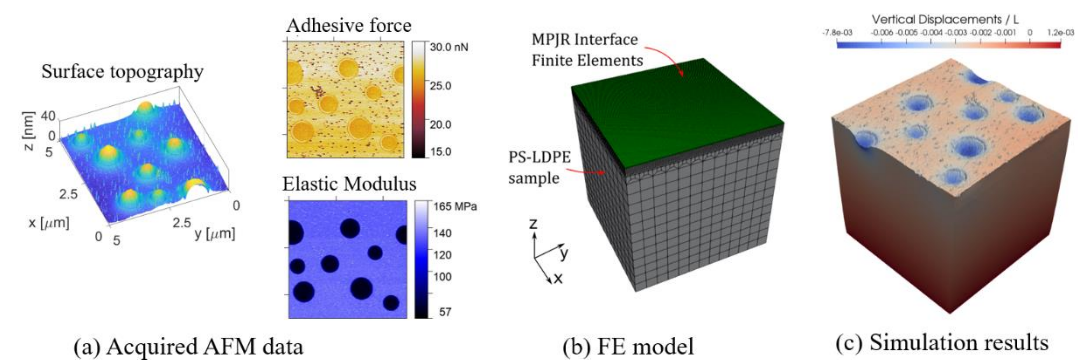

Figure 1: Simulation of a PS–LDPE sample accounting for surface topography, surface adhesion variability, and bulk material heterogeneity using the MPJR interface finite elements.

Acknowledgements
The authors thank support from the ERC-ADG-2021-101052956-BEYOND.

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Local Effects in Cylindrical Shells

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Keywords: Cylindrical shell, Vlasov's approach, Local effects

On the basis of Vlasov-like semi-membrane cylindrical shell solutions, which were very popular at the end of the 20th century and are now the subject of a renewed interest [1-2] due to the development of the GBT (Generalized Beam Theory) approach, some solutions are developed for the estimation of local effects in cylindrical shells. The goal of the study is to offer some accurate analytical formulae for the evaluation of local effects following imposed or prevented ovalisation upon bending. These effects, which may appear at first sight anomalous, have previously been noticed in the framework of experimental testing in offshore engineering, see Figure 1, and successively explained using both Finite Element analyses and approximate analytical formulations [3-5].

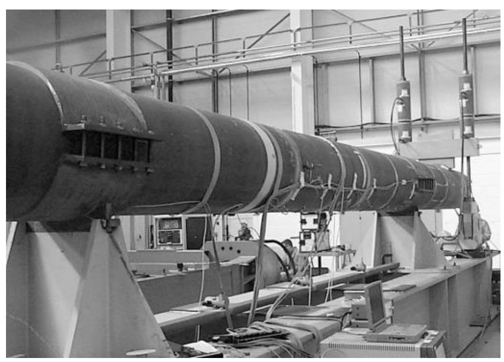

Figure 1: bending test arrangement for an offshore pipeline.

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Dynamic instability and limit cycles in nonlinear mechanical systems under non-holonomic constraints

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Keywords: Non-holonomic constraints, Dynamic instability, Method of Multiple Scales

Dynamic instabilities in elastic systems are traditionally linked to non-conservative follower loads, which also introduce counterintuitive phenomena, such as the well-known damping destabilization paradox observed in the Ziegler's double pendulum, where the system critical load decreases in the presence of small damping. Recent investigations [1] have shown similar dynamic instabilities and damping-induced paradoxical effects in conservative systems under non-holonomic constraints.

This study investigates the intricate interaction between nonlinear dynamics and non-holonomic constraints in non-linear elastic systems subjected to conservative loads. The analysis is based on a representative model (Fig.1): a double pendulum with a conservative loading mechanism at one end and a non-sliding wheel constraint at the other. Using the Method of Multiple Scales (MSM) as outlined in [2], limit cycles near Hopf bifurcations are determined analytically, providing new insights into the nonlinear dynamics induced by this configuration. The analytical results confirm that Hopf bifurcations can arise in conservative elastic systems when non-holonomic constraints are present. Notably, all perturbation equations of the MSM exhibit the same structure as those of the Ziegler's double pendulum, with key distinctions in the forcing terms, which here reinstate the system's conservative character. Numerical simulations (NUM) confirm the analytical findings.

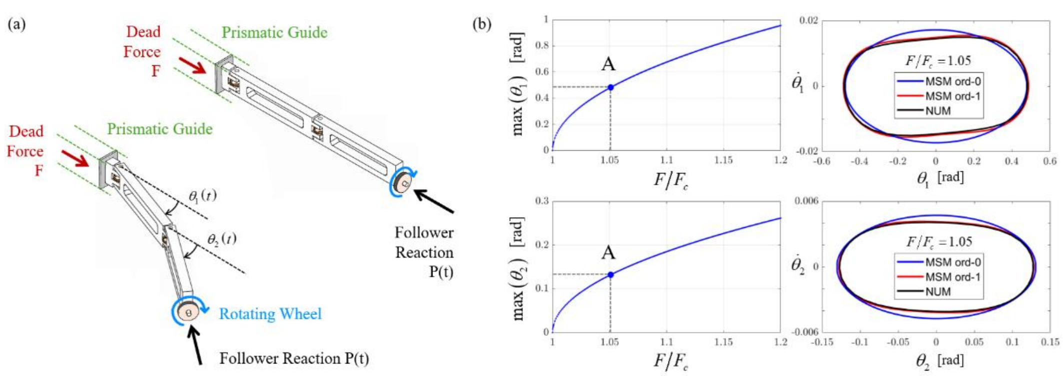

Figure 1: (a) schematic of the system and (b) limit cycle for F equal to 1.05 its critical value F_c, with indication of the point A in the bifurcation diagrams corresponding to $F/F_c = 1.05$.

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Screened Poisson Equation for Contact Detection in π-BEM

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Keywords: Approximate distance function, Contact Algorithms, BEM

Traditional contact detection algorithms for accelerated search of the contact region heavily rely on computational geometry. As an example, *slave-master* (node-to-segment) algorithms compute the gap between contacting surfaces via geometric projections. This approach may suffer from mesh dependencies and discontinuities resulting from nodes sliding between edges. More generally, the number of operations for contact search is proportional to the square of the number of cells on the boundary. Thus if the problem needs fine grid discretization on the boundary, it would also require a substantial computational effort to be solved.
As an alternative to algorithms that heavily rely on computational geometry, Areias et al. [1] use an Approximate Distance Function (ADF) obtained through the solution of the screened Poisson equation and a variable transformation. At the cost of solving an additional PDE they are able to obtain a much simpler algorithm to solve non-smooth contact problems in a finite element framework.
In this contribution, we implement the resolution of the screened Poisson equation in π-BEM, a parallel boundary element method solver developed by Mola et al. [2]. This is a first step towards a fully BEM-based contact formulation with a continuous gap function. Avoiding the discretization of the bulk of the bodies, this method could be more computationally efficient and preferable over the finite element method. In this regard, a future implementation of the Fast Multipole Method (FMM) for the screened Poisson equation within π-BEM could be considered to reduce computational costs.

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Imperfection sensitivity in stiffened cylindrical shells

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Keywords: Cylindrical shells, stiffened shells, buckling, imperfection sensitivity, knockdown factor, finite element simulations, elasticity

The elastic buckling of cylindrical shells under axial compression (or spherical shells under external pressure) is sudden and catastrophic, often driven by geometric imperfections orders of magnitude smaller than the shell radius. Research originating at JPL in the 1940's and continuing into the last decade has provided strong evidence that imperfections are the primary cause for the large discrepancies between experimentally observed and theoretically predicted values of the buckling load in historical datasets, and that precision characterization of imperfection geometry is required to accurately predict a shell's buckling load [1, 2]. However, despite recent advances in understanding and their widespread usage throughout the aerospace industry, there remains a clear lack of understanding within the community as to which types of imperfections are the most deleterious and how susceptible state-of-the-art reinforced shells, or stiffened shells, are to the most common types of imperfections. Merging precision machining with fluid coating techniques, we fabricate near-perfect and imperfect stiffened cylindrical shells from tailored elastomers with an industry standard stiffening geometry known as the isogrid pattern (Fig. 1). Designing our shells such that local buckling of the shell skin and stiffeners themselves are excluded, we show that such shells can be modeled with fidelity by the results for monocoque (uniform, unstiffened) shells having isotropic effective stretching and bending stiffnesses readily derived in terms of the shell skin and stiffener properties, where one of the most important parameters of the stiffened shell relevant to assessing the degrading role of imperfections on the buckling is the effective thickness, t_{eff}. Combining precision experiments with highly detailed finite element simulations, we show that isogrid shells with unrealistically large imperfections (on the order of t_{eff}), like their monocoque counterparts, buckle at loads that are a small fraction of the classical buckling load. Conversely, we show that iso-grid shells manufactured with both global and local imperfections of relatively small amplitudes, i.e. $\sim 0.1 t_{eff}$, have buckling knockdown factors significantly larger than those for their monocoque counterparts.

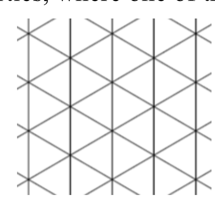

Figure 1. Isogrid pattern.

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Asymmetric and anisotropic elasticity of contact-based architected materials

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Keywords: elastic asymmetry, asymmetric anisotropy, architected materials, contact

Architected materials with internal contacts [1] represent a relatively novel and still underexplored class of materials. In addition to exhibiting specific dissipative properties and path-dependent behavior, these materials possess a distinctive feature: elastic asymmetry [2], i.e., the dependence of elastic constants on the sign of deformation. The organization and orientation of contact interfaces make these materials not only asymmetric but also anisotropic. This combination of asymmetry and anisotropy poses significant challenges in constructing associated homogenized models. However, understanding their behavior can enable intelligent exploration of such properties at the structural scale. Specifically, the unique properties of anisotropic asymmetric - or asymmetrically anisotropic -materials can be utilized to design structures with tailored stress distributions. Furthermore, constructing accurate constitutive models may lead to advancements in continuum damage mechanics, particularly for materials for which the primary source of damage arises from oriented cracks.

Currently, there is no consensus in the existing constitutive models [3,4] on how to effectively account for this interplay of asymmetry and anisotropy. To address this question, our direct numerical homogenization approach, incorporating accurate modeling of contact interactions, enables a critical analysis of existing constitutive models and facilitates the development of a new thermodynamically consistent framework.

The identified constitutive model will be applied to analyze classical solid mechanics problems in both static and dynamic contexts, including a bar under tension/compression, bending, and torsion, as well as a plate with a hole and a notched plate.

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