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Dynamic Instability and Limit Cycles in Nonlinear Mechanical Systems under Non-Holonomic Constraints

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Motivation and Background

Dynamic instabilities in mechanical systems have traditionally been linked to follower loads, which can also give rise to counterintuitive behaviors, such as the damping-induced destabilization in the Ziegler's double pendulum and Beck's beam, where the introduction of small internal damping reduces the critical load (Ziegler's paradox). Similar destabilization phenomena occur in systems with nonlinear damping, where internal and external nonlinear damping sources can destabilize otherwise stable limit cycles, leading to a hard loss of stability [1,2]. Flutter and divergence instabilities can also be triggered by dry friction (Fig. 1), as demonstrated experimentally by Bigoni and Noselli [3]. Recently, studies have identified analogous instability mechanisms and damping-induced destabilization in conservative systems subjected to non-holonomic constraints [4]. This study provides an analytical exploration of the intricate interaction between nonlinear dynamics and non-holonomic constraints in nonlinear mechanical systems subjected to conservative loading.

Methods and Results

The analysis centers on a representative model (Fig. 2): a double pendulum with a conservative loading mechanism at one end and a rolling wheel at the other. The governing equation is in the form:

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{C}\dot{\mathbf{q}}(t)+[\mathbf{K}+\mu\mathbf{H}]\mathbf{q}(t)=\mathbf{n}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}},\mu)$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, $\mathbf{H}$ are constant matrices, $\mathbf{n}$ is a vector of nonlinearities, $\mathbf{q} = [\theta_1, \theta_2]^{\mathrm{T}}$ is a vector of independent Lagrangian parameters, and μ denotes the conservative load.

Employing the Method of Multiple Scales (MSM) as in [2], limit cycles near Hopf bifurcations are derived analytically, providing new insights into the nonlinear dynamics induced by this configuration. Remarkably, the perturbation equations obtained via MSM share the same structure as those of Ziegler's double pendulum, with key distinctions in the forcing terms, which here restore the system's conservative nature. The analytical findings demonstrate that Hopf bifurcations can occur in conservative systems when non-holonomic constraints are present. Moreover, the presence of damping can negatively impact not only the critical load that triggers the bifurcation but also the post-critical dynamics and the stability of the resulting limit cycles. Numerical simulations (e.g., Fig. 3) confirm the analytical findings.

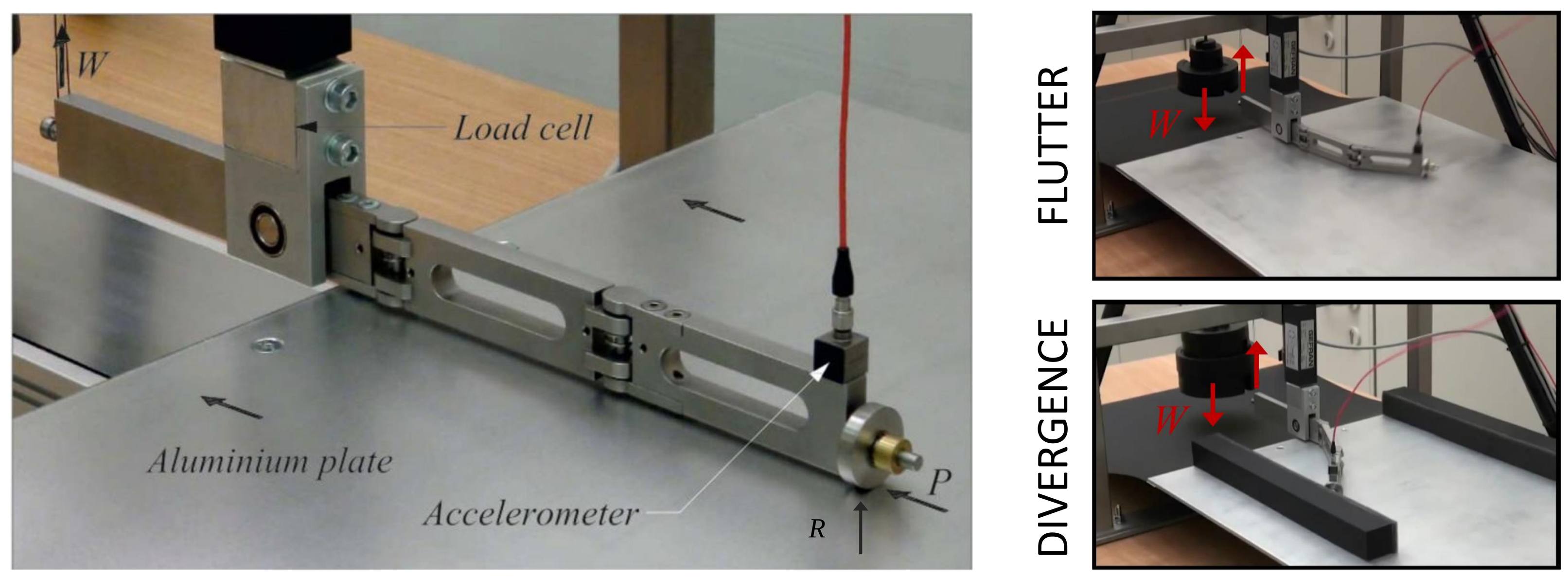

Figure 1: Double pendulum mounted on a moving plate, subjected at one end to a frictional force **P**, which is proportional to a normal reaction, **R**, generated by a suspended weight, **W**, through a lever mechanism.

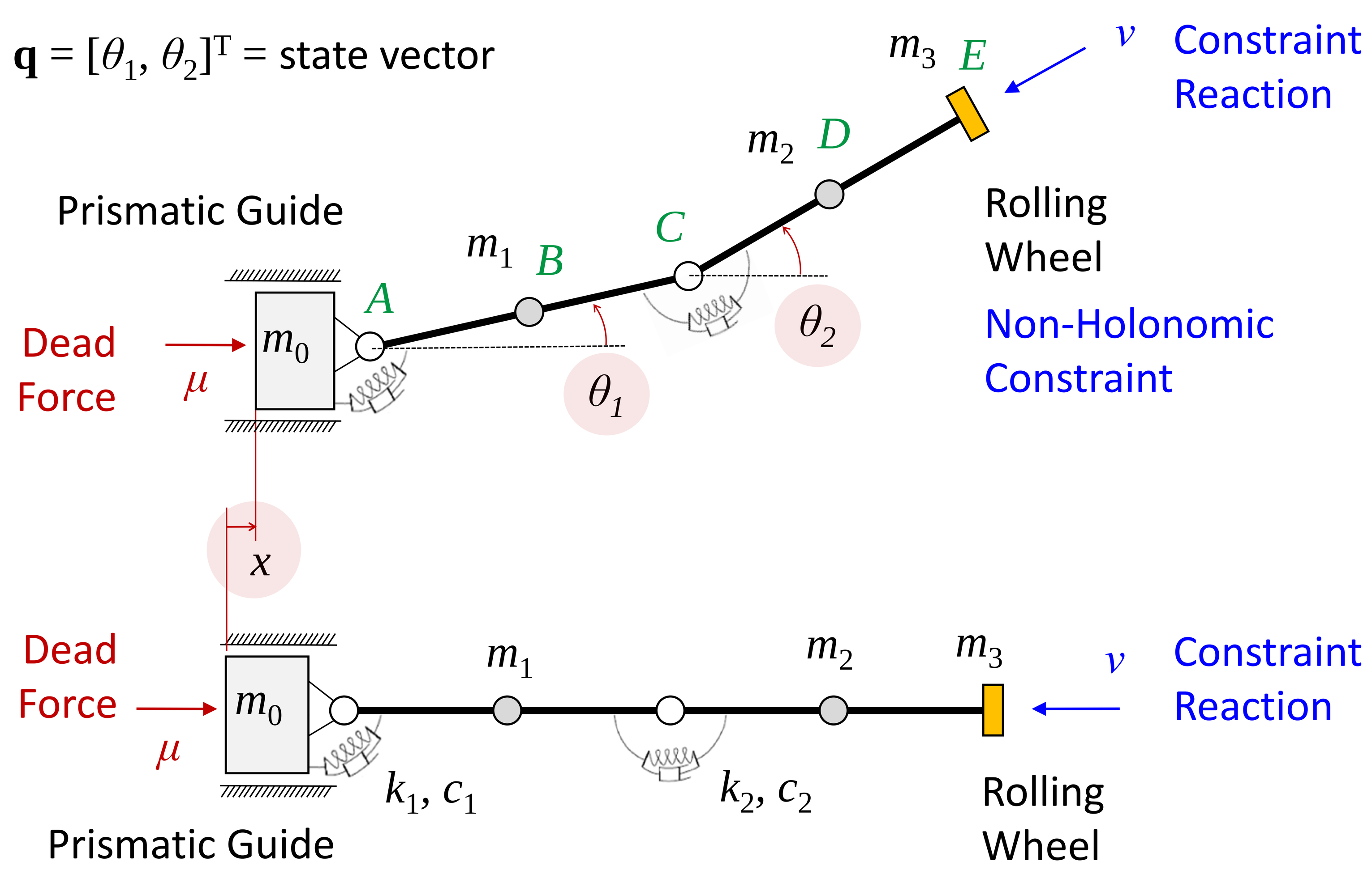

Figure 2: Non-holonomic double pendulum derived from the system in Fig. 1 by fixing the moving plate and introducing a prismatic guide together with a dead force μ that acts through the guide, pushing the system.

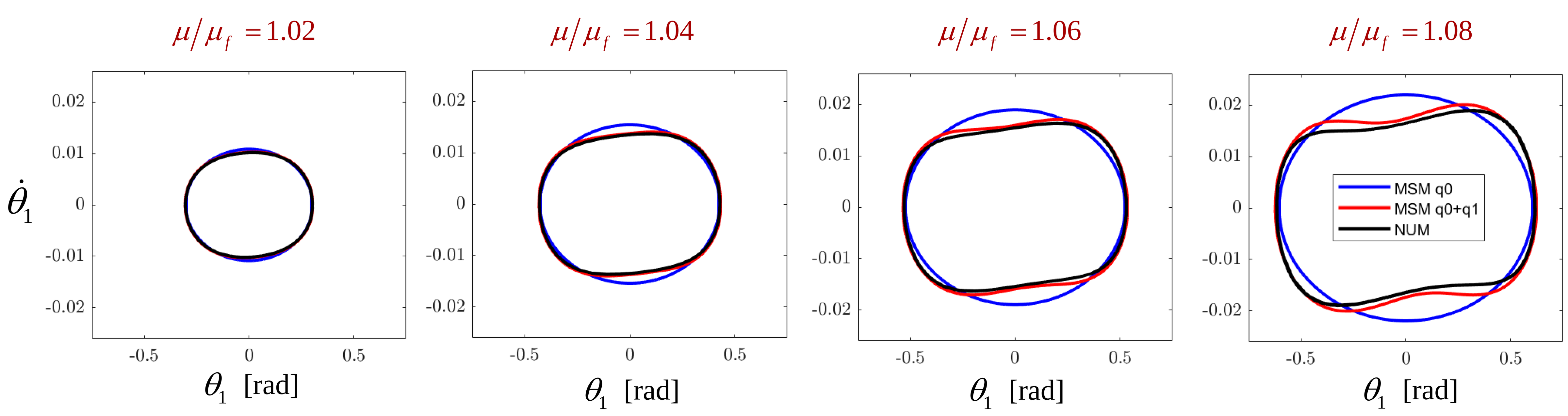

Figure 3: Analytical solution (MSM orders 0 and 1) and numerical solution (NUM) providing the phase portrait of the limit cycle of the lagrangian variable θ_1 and its maximum value as a function of the conservative load parameter μ measured from the Hopf critical load μ_f. Similar results hold for the lagrangian variable θ_2.

References

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