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Title

Dynamic instability of nonlinear mechanical systems subject to non-holonomic constraints

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Abstract

The complex interplay between nonlinear dynamics, stability, and non-holonomic constraints in elastic systems is explored through the paradigmatic model of a double pendulum. This system is subject to a conservative force at one end and a non-sliding wheel constraint at the other. Interestingly, and in contrast to the well-known Ziegler's double pendulum with a non-conservative follower force, the studied system can exhibit dynamic instabilities such as flutter and divergence even under purely conservative loading conditions. Dynamic instabilities in elastic systems are traditionally linked to non-conservative follower loads, which also introduce counterintuitive phenomena. One such phenomenon is the well-known "Ziegler's paradox", or damping destabilization paradox, where the critical load of the system decreases when small damping is present, compared to the undamped case. Typically, instabilities like flutter and divergence, as well as paradoxical effects such as damping-induced destabilization, are regarded as exclusive to systems under non-conservative loading conditions. However, recent investigations suggest that similar behaviors can also arise under conservative loading conditions when non-holonomic constraints are introduced [1]. This highlights the crucial role of non-holonomic constraints in influencing the nonlinear dynamic behavior and stability of elastic systems.

This study analytically examines the nonlinear dynamics of a double pendulum with a non-holonomic constraint at one end and a conservative loading device at the other. Using the Method of Multiple Scales as in [2], the analytical determination of the limit cycles of the system near Hopf's bifurcation points is achieved, offering new insights into the nonlinear dynamics induced by this configuration. The findings reveal that Hopf's bifurcations can emerge in elastic systems under purely conservative loading conditions when non-holonomic constraints are present. Interestingly, the perturbation equations at different orders of the Multiple Scales Method exhibit the same non-symmetric structure as those governing the Ziegler's double pendulum, except for the force term, which restores the conservative nature of the system under study. The amplitude of the resulting limit cycles is derived analytically and corroborated through comparisons with benchmark numerical solutions. This analytical framework opens avenues for designing advanced mechanical systems with potential applications in areas such as limbless locomotion, where non-holonomic constraints and dynamic stability play crucial roles.

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