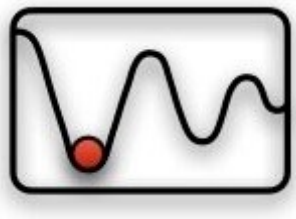
XXIII International Conference on Waves and Stability in Continuous Media

WASCOM 2025

Parma, 9-13 June 2025



Book of abstract

Conference Information

The **International Conference on Waves and Stability in Continuous Media** (WASCOM), now in its XXIII edition, is a biennial international event focused on Mathematical Physics. Since its first edition in 1981, WASCOM has provided a valuable platform for both Italian and international researchers to engage and collaborate on topics related to stability and wave propagation in continuous media.

The conference covers a broad range of research areas, including but not limited to: shock waves, diffusion processes in biology and continuum mechanics, kinetic models, non-equilibrium thermodynamics, stochastic processes, group theoretical methods, and advanced numerical techniques.

Main Topics of the Conference

- Linear and nonlinear stability in fluid dynamics and solid mechanics
- Non-linear wave propagation, discontinuity and shock waves
- Rational extended thermodynamics and symmetric hyperbolic systems
- Kinetic theories and comparison with continuum model
- Numerical applications

The previous editions of the conferences were organized in **Catania** (1981), **Cosenza** (1983), **Bari** (1985), **Taormina** (1987), **Sorrento** (1989), **Acireale** (1991), **Bologna** (1993), **Palermo** (1995), **Bari** (1997), **Vulcano** (1999), **Porto Ercole** (2001), **Villasimius** (2003), **Acireale** (2005), **Scicli** (2007), **Mondello** (2009), **Brindisi** (2011), **Levico** (2013), **Cetraro** (2015), **Bologna** (2017), **Maiori** (2019), **Catania** (2022), **Bari** (2023).

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Abstracts

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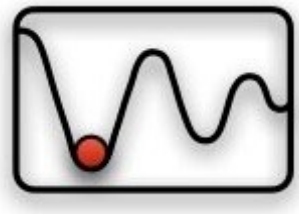

Role of cross-diffusion in pattern formation for a modified Klausmeier-Gray-Scott model

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Spatial vegetation patterns, such as bands, spots, and gaps, are prominent characteristics of landscapes found in many semi-arid regions. Approximately 30% of the emerged surface of the earth is covered with these patterns. Between shrubs, grass, bushes or trees empty spaces appear at regular intervals. On hillsides, strips of vegetation alternate with strips of bare ground in a direction parallel to the hill's contours. On flat ground, a wide diversity of stationary patterns occurs.

Here, we consider the pattern formation in a diffusive Klausmeier model [1] where the plants grow on flat land instead of hill. It is worth noting that the corresponding kinetic part of the model we considered also arises from an autocatalytic chemical reaction model first proposed by Gray and Scott [2, 3], so the system we are studying is also called diffusive Klausmeier-Gray-Scott model. In this model, the advection term becomes negligible and the dominant evolutionary mechanism is modelled by a Laplacian. Moreover, inclusion of a nonlinear cross-diffusion term allows to model the dynamics of the biomass influenced by the presence of the water, adding a biomass flux which is proportional to the gradients of the water concentration and to biomass itself. When the cross-diffusion coefficient is negative, the biomass tends to move towards regions with higher water concentration, and the opposite behavior occurs when the coefficient is positive.

To better evaluate the effect of the new nonlinear cross-diffusion term, and reduce the mathematical complexity of the problem, we will consider a one-dimensional version of the model, assuming that all functions be space-homogeneous in one of the two space variables and removing the drift term in the equation for the water density, which in the original Klausmeier model plays a crucial role for the pattern formation. Our main result is that also the cross-diffusion term can cause Turing instability and, thus, generation of patterns, similar to the ones caused by the presence of the drift term.

Finally, a weakly nonlinear analysis around the critical value of the cross-diffusion is performed, as in [2] for a predator-prey system with cross-diffusion, and the asymptotic expansion is validated by numerical solution of the full system.

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Nonlinear damping in large-amplitude vibrations

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An increase in damping is relevant for the passive control of vibrations and noise; therefore, it is very significant in design. Experimental data show a strong and nonlinear dependence of damping on the vibration amplitude for beams, plates, and shells of different sizes and made of different materials (metal, composite materials, silicone rubber, and graphene). While the frequency shift of resonances due to stiffness nonlinearity is commonly 10 to 25% at most for common structural elements, a damping value up to several times larger than the linear one can be obtained for vibrations of thin plates when the vibration amplitude is about twice the thickness [1, 2, 3]. This is a huge change in the damping value! Therefore, the nonlinear nature of damping affects structural vibrations much more than stiffness nonlinearity. Despite this experimental evidence, nonlinear damping has not been sufficiently studied yet. A model of nonlinear damping was derived from linear viscoelasticity for single-degree-of-freedom systems and for rectangular plates by introducing geometric nonlinearity [1, 2, 3]. The resulting damping model was nonlinear, and the model parameters were identified from experiments. Numerical results for forced vibration responses of different structural elements in large-amplitude (nonlinear) regimes were obtained and successfully compared to experimental results, validating the nonlinear damping model. Recently, the effect of nonlinear damping on one-to-one internal resonances was addressed. This type of resonance appears in the case of symmetry: circular cylindrical shells and square plates are examples, as well as beams with circular cross-section in three-dimensional space. The results of this new development in the study of nonlinear damping are particularly interesting.

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Solitary wave propagation in tensegrity lattices

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The development of noninvasive techniques for detecting material defects and monitoring structural health is an emerging research topic in several engineering disciplines. Traditional approaches to acoustic wave focusing - largely based on linear acoustic effects - often face limitations related to tunability and scalability. In contrast, strongly nonlinear systems offer enhanced control over wave speed, potentially enabling the design of novel devices with superior focusing capabilities.

In this work, we explore the potential of tensegrity-based metamaterials with nonlinear elastic behavior to design innovative systems for impact mitigation, tunable acoustic lenses, and structural health monitoring tools. These metamaterials support the propagation of both compression and rarefaction solitary waves. Considering previous studies in one-dimensional (1D) settings, we extend the investigation to two-dimensional (2D) and three-dimensional (3D) tensegrity beams and plates, which function as phononic crystals. An analytical model is also developed to describe the existence and features of solitary wave dynamics in the 1D case and in a continuum media.

[1] Fraternali, F., Senatore, F., Daraio, C.: Solitary waves on tensegrity lattices. *J. Mech. Phys. Solids*, **60**, 1137–1144 (2012)

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New kinetic boundary conditions and their applications

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In recent papers [1, 2, 3], a kinetic equation for gas-surface interaction was proposed and was applied to the construction of the boundary condition for the Boltzmann equation. The kinetic equation includes an attractive-repulsive potential appreciable only in the vicinity of the solid surface and growing indefinitely on the surface. This potential forms a thin layer of physisorbed molecules (physisorbate layer). The kinetic equation also includes the interaction term between gas molecules (the Boltzmann collision term) and that between gas molecules and phonons. It was assumed that: (i) the gas-phonon interaction was dominant compared with the gas-gas interaction inside the physisorbate layer; (ii) the thickness of the physisorbate layer was much smaller than the mean free path of the gas molecules; and (iii) the gas-phonon interaction was described by a simple collision model of relaxation type. Under these assumptions, an asymptotic analysis was performed, and a linear kinetic equation for the physisorbate layer was derived. The resulting kinetic equation and the boundary condition at infinity form a half-space problem.

The point is that the solution of the half-space problem for the physisorbate layer establishes the boundary condition for the Boltzmann equation on the boundary. To be more specific, if the velocity distribution for the incoming molecules at infinity is specified, that for the outgoing molecules at infinity is determined. The former and latter distributions correspond, respectively, to the velocity distribution for the incident molecules and that for the reflected molecules on the boundary for the Boltzmann equation, which describes the behavior of the gas outside the (infinitesimally) thin physisorbate layer. This property, i.e., the well-posedness of the physisorbate-layer problem, was confirmed numerically in [1] and then proved rigorously in [2]. However, it is practically impossible to obtain the solution analytically in an explicit form. Therefore, an iteration scheme was devised, and the approximate solutions based on the first and second iterations were obtained explicitly. From these explicit approximate solutions, new models of the boundary condition for the Boltzmann equation have been established [1, 2, 3]. The first-iteration model is a Maxwell-type condition with a velocity dependent accommodation coefficient, and the second-iteration model has a diffusive part with more complex structure.

In this talk, we first summarize the new models of the boundary condition and their properties. Then, we apply the models to some fundamental problems of rarefied gas dynamics, such as the heat transfer between two plates and plane Couette flow, and discuss the numerical results.

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An ES-BGK model incorporating molecular rotational and vibrational modes

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The ellipsoidal statistical (ES)-BGK model proposed in [1, 2] is widely used to describe collision processes in kinetic models of rarefied polyatomic gases. However, it encounters limitations when applied to non-polytropic polyatomic gases, whose specific heat varies with temperature. To address this challenge, in [3] we developed a novel ES-BGK-based kinetic model for non-polytropic gases within the framework of the Borgnakke–Larsen kinetic model [4], where molecular internal degrees of freedom are represented through continuous internal-energy parameters.

In certain polyatomic gases, such as carbon dioxide, molecular vibrational relaxation occurs considerably more slowly than rotational relaxation, necessitating distinct kinetic treatments for these modes. Although the generalized BGK model [5], which is aligned with macroscopic phenomenological energy-relaxation equations, has been established, the corresponding ES-BGK model featuring separate rotational and vibrational energy modes remains largely undeveloped, apart from discrete vibrational-energy treatments with some limitations examined by [6, 7].

In this study, we propose an advanced ES-BGK model that explicitly incorporates separate molecular rotational and vibrational degrees of freedom using continuous internal-energy parameters. This model offers a more accurate and flexible description of non-equilibrium phenomena in polyatomic gases, significantly extending the applicability and predictive capability of the ES-BGK approach.

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Interfacial instability in two-phase flow dynamics

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Carbon dioxide is a cost-effective, reliable, and environmentally friendly refrigerant that is increasingly used in evaporator design. A thorough understanding of the underlying flow physics, along with accurate predictions of phase change through boiling, is crucial for the widespread adoption of CO2 as a coolant. Among the various physical phenomena involved, dryout inception is particularly critical because it marks the transition from annular to mist flow. This transition associated with a sudden drop in the heat transfer coefficient, which can have potentially catastrophic consequences. Two opposing behaviors regarding the onset of dryout, from now on defined as δ^- and δ^+ regimes, have been reported in [1]. The δ^- regime is characterized by a decreasing dryout vapor quality with increasing mass flux, whereas the δ^+ regime - more relevant for CO2 in microchannels - exhibits an increasing dryout vapor quality with mass flux. A detailed experimental campaign conducted at CERN has provided unprecedented insight into the mechanisms leading to dryout inception.

In this presentation, I will introduce a mathematical model developed to predict the instability of the smooth liquid-vapor interface characterizing annular flow. This model is motivated by the ansatz that *dryout occurs due to the disruption of the interface caused by interfacial instability.*

Within the framework of continuum mechanics, we derive the mathematical model by imposing the conservation of momentum and mass astride the interface. Heat exchange between liquid and vapor has been modelled through a density production term (Hsieh's hypothesis, [2]), while the normal component of the stress tensor at the interface was described by the Young-Laplace equation. Once the interfacial conditions were obtained along with the governing equations in dimensionless form, we have performed an instability analysis, introducing a key dimensionless group characterizing the onset of dryout. Numerical investigation show that the correlation between mass flux and dryout vapor quality is consistent with the experimental data, validating the hypothesis that dryout inception is governed by interface instabilities.

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Assessing respiratory virus co-infections using an identifiable model: the case of influenza and SARS-CoV-2 in Italy

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The study investigates the dynamics of co-infections during an epidemic, particularly in the absence of official data on co-infected individuals.

In the first part of the research the primary objective is to assess the robustness of the two-pathogen co-infection model proposed by Fahlena et al. [1] in terms of both structural and practical identifiability [2, 3, 4]. This analysis is based on official data regarding new influenza and SARS-CoV-2 cases in Italy during the winter of 2023-2024, complemented by estimated co-infection data under two scenarios (high and low levels of co-infection). The study finds that when both weekly infection and co-infection data are available, the model's parameters are structurally identifiable. However, if only incidence data for each virus are available, five parameters must be fixed to achieve the structural identifiability of the remaining parameters. Under these hypotheses, the model results to be also practically identifiable.

The second part of the research aims to evaluate the time variation of the co-infection percentages in Italy during the study period. The model suggests that a unimodal time profile of co-infection percentages could have occurred in this situation. These results emphasize the importance of comprehensive data for model identification and co-infection estimation during epidemics.

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Half-space problems for the Boltzmann equation for polyatomic gases

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Half-space problems in the kinetic theory of gases are of great importance in the study of the asymptotic behaviour of solutions of boundary value problems for the Boltzmann equation for small Knudsen numbers. They provide the boundary conditions for the fluid-dynamic-type equations and Knudsen-layer corrections to the solution of the fluid-dynamic-type equations in a neighbourhood of the boundary. These problems are well-studied for monatomic species, especially for single, but to some extent also for multicomponent, gases. It is well-known that the number of additional conditions needed to be imposed depends on different regimes for the Mach number (corresponding to subsonic/supersonic evaporation/condensation). The case of polyatomic species is not as well studied in the literature.

In this talk, we will discuss some extensions of the results for half-space problems for monatomic gases to the case of polyatomic gases.

Self-origami with structures & solids

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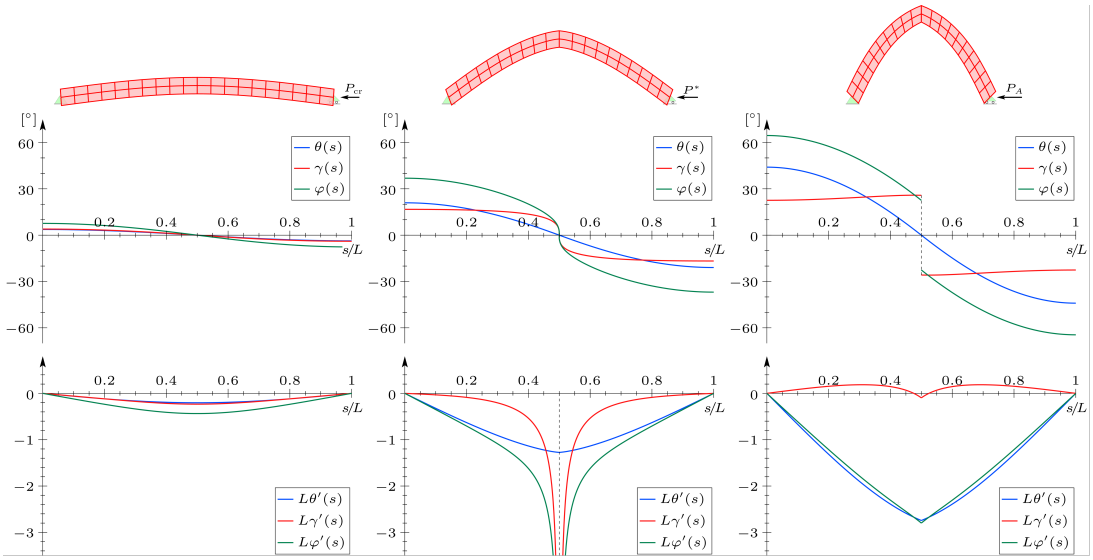
Folding is a process in which curvature localizes in space and progressively forms sharp edges, separated by almost undeformed elements. Origami is created by inducing folding in thin films, which can be made from paper or other materials. Differently from the creation of origami, folding in rock formations, marine shells, or other natural elements is a spontaneous process, driven by forces and, for living matter, growth. The purpose of the presentation is to develop the modelling of spontaneous folding in structures and solids. A new continuous model of shearable rod, subject to large elastic deformation, is derived from nonlinear homogenization of a one-dimensional periodic microstructured chain. As particular cases, the governing equations reduce to the Euler elastica and to the shearable elastica known as 'Engesser'. The postcritical response of the simply supported equivalent continuous rod exhibits the emergence of folding, Fig. 1, an infinite curvature occurring at a point of the rod axis, developing into a curvature jump at increasing load[1]. In solids, folding is shown to occur in an elastic solid obeying the couple-stress theory with extreme anisotropy[2].



Acknowledgments: Financial support from ERC-ADG-2021-101052956-BEYOND.

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Multiscale models predicting crack nucleation and propagation under thermal and rate effects

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Fracture and decohesion phenomena are widely studied across various disciplines due to their fundamental theoretical significance and broad range of applications. One of the most established and widely employed frameworks in this field is the energetic approach, encapsulated by Griffith's energy criterion. While analytical and experimental studies have confirmed the effectiveness of this criterion in describing the propagation of pre-existing cracks, it remains inadequate for capturing several critical aspects relevant to structural design. Notably, it fails to predict crack nucleation and presents significant challenges in rigorously incorporating temperature effects within an analytical framework.

This study investigates the influence of temperature on crack nucleation and propagation in material failure and decohesion. Building upon Griffith's energy criterion, we propose a simplified model focused on mode I fracture, extending the classical criterion to account for crack nucleation and the role of thermal fluctuations. By leveraging tools from equilibrium statistical mechanics, we integrate entropic contributions into the overall energy balance. Additionally, we adopt a multiscale approach, simultaneously formulating both discrete and continuum (limit) models. This methodology provides deeper insight into the intricate mechanisms governing fracture and decohesion, elucidating how microscopic-scale phenomena influence meso- and macroscopic behavior.

Despite the simplicity of the proposed models, they allow for analytical tractability and a more pro- found understanding of the underlying physics. Our energetic approach effectively captures the competition among external loading, elastic deformation, fracture energy, and entropic effects. Specifically, our model predicts crack nucleation and quantifies the influence of thermal fluctuations on this process. Furthermore, the framework accommodates different fracture and decohesion scenarios, including cases where fracture propagates from one end or where the damaged region remains confined within the system, such as in the presence of multiple bubbles (e.g., DNA denaturation bubbles). In the latter case, our model predicts the coalescence of these bubbles prior to complete failure.

Interestingly, our approach uncovers a classical critical behavior, wherein the critical load decreases with increasing temperature following the relation $(1 - T/Tc)^{1/2}$. Consequently, at the critical temperature Tc, the system undergoes a phase transition, leading to complete rupture even in the absence of an applied mechanical load.

In addition to temperature effects, our prototypical model also captures rate-dependent behavior, i.e., the system's response under a time-dependent loading rate. Unlike temperature effects, which we analyze using equilibrium statistical mechanics, rate effects necessitate a departure from equilibrium conditions. Preliminary results indicate that the system's dynamics are significantly influenced by the applied loading rate. Specifically, as the loading rate increases, the fracture propagation velocity also increases, whereas a

decrease in the loading rate leads to a softening of the system's mechanical response.

The findings regarding both temperature and rate effects are particularly promising, as they align with experimentally observed behaviors in materials that have been challenging to describe rigorously through analytical approaches. This study introduces a relatively simple yet powerful model that captures these complex phenomena without compromising mathematical tractability and analytical rigor.

Quasi-resonant collisions: a kinetic setting for bi-temperature modeling

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[3]*IRMAR, Université de Rennes, Rennes, France*
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The study of polyatomic gases in kinetic theory has been an active field of research during the past decades. Due to rotation and vibration mechanisms, polyatomic molecules have an internal structure that influences the global dynamic.

Some molecules, like CO_2, exhibit a peculiar behavior during collisions called *resonant*: they exchange *separately* kinetic and internal energies. A resonant Boltzmann model was constructed by Boudin *et al.* in [1], where they showed that the resonant equilibrium distribution has two separate temperatures, internal and kinetic, as there are no exchanges between these two parts.

Following this line, we propose in [2] a Boltzmann model in which molecules undergo *quasi-resonant* collisions: they exchange *almost separately* kinetic and internal energies. While approximating resonant interactions, we show that the quasi-resonant equilibrium distribution has a single temperature.

We therefore argue that the quasi-resonant dynamic is a two-phase relaxation process: an first resonant-like relaxation towards a two-temperature Maxwellian state, followed by a relaxation of the two temperatures towards each other (where the distribution stays of two-temperature Maxwellian shape) which is of Landau-Teller type and that we derive explicitly for some specific cross-sections. Numerical simulations validate our theoretical predictions.

The quasi-resonant setting is therefore a potentially useful kinetic model for the study of bi-temperature systems.

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A kinetic model for opinion dynamics driven by individuals' variable activity level

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We present a kinetic model for opinion dynamics that incorporates the propensity of individuals to engage in social interactions, represented by a variable activity level. The population is divided into three groups: active agents, who are socially engaged and likely to interact; inactive agents, who have a low probability of interaction; undecided agents, who serve as intermediaries between the other two groups.

The equilibria of the model reveal that an effective compromise process among active agents fosters consensus formation, preventing the emergence of extreme opinions within this group. Conversely, the absence of such process among inactive individuals can lead to the development of extreme opinions among them, resulting in opinion polarization.

Finally, we introduce a simple but effective control strategy to increase the number of active individuals and reduce the number of inactive ones. We also provide a convergence to equilibrium result for the controlled model, demonstrating that opinion polarization can be entirely mitigated within the population.

A rational extended thermodynamics perspective on bubble dynamics

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Since the beginning of the 20th century, with Lord Rayleigh's work on the collapse of a cavity in a fluid, the mathematical modelling of gas bubbles in a liquid has become a crucial and challenging task, with a view to understanding their physical behaviour and improving their countless practical applications [1, 2, 3]. In order to obtain analytical and numerical models that are not too computationally expensive, one commonly starts from an oversimplification of the description, with particular regard to gas dynamics. However, it is well-known that this can lead to incorrect results, unable to provide accurate predictions [3]. Speaking of which, it is worth recalling, for example, the importance of thermodynamic effects [4], often neglected to reduce the complexity and the number of equations to be solved.

A further difficulty is presented by the impossibility of having direct measurements of the physical quantities related to the gas in the bubble (mass density, pressure, temperature) due to the small physical dimensions and the extreme rapidity of the phenomena involved. Usually, what is observable and measurable is the overall effect related to the oscillation of the gas volume inside the bubble or its dissolution under the action of a sound signal.

To change point of view and better understand which elements can significantly influence bubble dynamics, we adopt a Rational Extended Thermodynamics (RET) perspective. RET theory [5, 6], as the name itself suggests, is a thermodynamic theory that extends the role of independent field variables to non-equilibrium quantities such as stress tensor, dynamic pressure and heat flux, with the aim of better describing non-equilibrium phenomena. Here we present the first results of this study, which have enabled us 1)to highlight the role of dynamic pressure [7], 2)to compare the models already used in literature (including the one that refers to the Navier-Stokes-Fourier approximations), 3)to investigate the range of validity of the approximations introduced by each approach, also through stability analysis.

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[3] Lauterborn, W., Kurz, T.: Physics of bubble oscillations. *Rep. Prog. Phys.*, **73**, 106501 (2010)

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Fast reaction limit for a class of triangular cross-diffusion systems

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In this talk, we analyse a class of cross-diffusion systems, modeling the evolution of densities of multicomponent populations in interaction, in the context of dietary diversity and starvation. More precisely, we derive a class of triangular cross-diffusion systems as its fast reaction limit. The main tools used are a family of entropy functionals and compactness arguments. However, we also investigate the regularity of the obtained solution, by improving the entropy a priori estimates, thanks to a bootstrap argument. We conclude the analysis with a weak-strong stability result. This talk is based on [1, 2].

This is a joint work with L. Corrias , L. Desvillettes and H. Dietert.

[1] Brocchieri, E., Corrias, L.: On a class of triangular cross-diffusion systems and its fast reaction approximation. *arxiv.org/abs/2503.07156* (2025) - Preprint.

[2] Brocchieri, E., Corrias, L., Dietert, H., Kim, Y-J.: Evolution of dietary diversity and a starvation driven cross-diffusion system as its singular limit. *J. Math. Biol.*, **83** (2021).

Behavioural integral epidemic models with information index

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Epidemics of infectious diseases are complex phenomena, influenced not only by biological factors but also by sociological and behavioural dynamics. In this talk, I will explore integral epidemiological models that combine the epidemiological aspects with human behaviour and consider the impact of information on social interactions. The models presented include an information index that simulates changes in people's behaviour in response to the evolving infection. Specifically, I will discuss how the integration of memory functions (such as Erlang kernels) can affect stability and oscillations in the models, with applications to diseases like influenza and SARS.

I will also discuss how ordinary differential equations, used in minimal models, can be extended to more complex integral models with information-dependent contact patterns. The asymptotic properties and stability conditions of these models will be examined, highlighting the effect of the memory kernel on epidemic dynamics. Numerical results demonstrate how the historical memory of infection can lead to significant dynamic behaviours, influencing the predictions of epidemiological models in real-world scenarios.

This research is conducted in collaboration with E. Messina and C. Panico from the University of Naples Federico II [1, 2, 3].

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[2] Buonomo, B., Messina, E., Panico, C., Vecchio, A.: An integral renewal equation approach to behavioural epidemic models with information index. *J. Math. Biol.*, **90**, 8(2025)

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An adjoint method for optimization of the Boltzmann equation

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We present an adjoint method for optimization of the spatially homogeneous Boltzmann equation for rarefied gas dynamics. The adjoint method is derived using a "discretize then optimize" approach. Discretization (in time and velocity) is via the Direct Simulation Monte Carlo (DSMC) method, and adjoint equations are derived from an augmented Lagrangian. After a forward (in time) solution of DSMC, the adjoint variables are found by a backwards solver. They are equal to velocity derivatives of an objective function, and are used for optimization of the Boltzmann equation. For general collision models, DSMC requires the use of a rejection sampling step, which involves discontinuities that lead to a new term, involving a "score function".

This is joint work with Yunan Yang (Cornell) and Denis Silantyev (U Colorado, Colorado Springs).

Asymptotic behaviour of throughflow solutions in the class of Jeffreys' fluids

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In the present talk, a consistent model for non-isothermal viscoelastic fluid [1] of Jeffreys type [2] forming a horizontal layer heated from below is introduced and the stability of a vertical constant throughflow [3] is analyzed. Planes delimiting the layer are assumed isothermal, rigid and permeable.

Via linear analysis, it is proved that the strength of the vertical throughflow affects the number of modes leading to the onset of oscillatory instability and that motions originating at the onset of instability are oscillating in time for strong enough throughflows, regardless the impact of the fading memory behavior. The instability analysis of the throughflow solution reveals that viscoelastic fluids with stronger elastic properties (i.e., exhibiting a more elastic response relative to a viscous one) are more prone to oscillatory instability. However, triggering the instability still requires a higher Rayleigh number, meaning that while the elastic component facilitates oscillatory instability, a greater thermal gradient (Rayleigh number) is still necessary for its onset. Moreover, a sufficient condition for nonlinear stability of the throughflow has been obtained, by introducing a suitable L^2-norm.

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Rings dynamics around small bodies

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It is widely known that the giant planets possess ring systems; it is less commonly known that certain minor bodies of the Solar system have rings. Remarkably, in most cases the rings of these minor bodies are located in a 1:3 Lindblad resonance, meaning that a ring particle completes one orbit for every three full rotations of the central body about its spin axis.

Using an appropriate Hamiltonian model expressed in terms of epicyclic variables, we apply perturbation theory and bifurcation theory to substantiate the selection of the 1:3 resonance for minor bodies, when compared to corotation and other Lindblad resonances. Our results highlight the role of the shape parameters, precisely oblateness and elongation, which are particularly relevant for minor bodies.

On variable-order fractional calculus and linear viscoelasticity

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In recent decades, physical phenomena characterized by long memory, such as non-exponential relaxation, and non-local effects have attracted considerable interest. Among the mathematical tools available to address these phenomena, fractional calculus has become a particularly favored method for modeling a broad spectrum of effects involving non-localities in both space and time. A fractional derivative extends the concept of an integer-order derivative to non-integer orders, allowing for more flexible and accurate modeling of processes that exhibit memory and hereditary properties [1].

Building on this, Giambattista Scarpi introduced an innovative approach to variable-order fractional calculus [2, 3], which adapts the order of differentiation as a function of time. This method provides a more nuanced and dynamic framework for modeling systems where the memory effects evolve over time, enhancing the applicability of fractional calculus in various complex scenarios. Specifically, given the variable-order function $\alpha : [0,T] \to (0,1)$, such that $t \mapsto \alpha(t)$, the Laplace transform $A(s) = \mathcal{L}\big(\alpha(t); s\big)$ defines $\Phi_\alpha(s) := s^{sA(s)-1}$. This latter expression, Laplace inverted in the time domain, defines the kernel $\phi_\alpha(t) := \mathcal{L}^{-1}\left(\Phi_\alpha(s)\,,\, t\right)$ that identifies the so called Scarpi's fractional derivative [1]

$$ {}^{\mathrm{C}}D_0^{\alpha(t)} f(t) = \int_0^t \phi_\alpha(t-\tau)\, f'(\tau)\, \mathrm{d}\tau\,. $$

In particular, I will discuss the application of Scarpi's variable-order fractional calculus to linear viscoelastic materials [4]. By employing a variable-order fractional Maxwell model,

$$ \sigma + a_1 D^{\alpha(t)} = b_1 D^{\alpha(t)} \varepsilon\,, \qquad a_1,\, b_1 > 0\,, $$

and I will show how the order of the fractional derivative can be modulated to capture the evolving viscoelastic behavior of materials under different loading conditions. This analysis includes both theoretical insights and numerical simulations, highlighting the advantages of this approach in accurately describing the time-dependent mechanical properties of viscoelastic materials. The results underscore the potential of variable-order fractional calculus as a powerful tool for advancing the modeling and understanding of complex physical systems with memory and hereditary characteristics.

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A kinetic model for international trade with knowledge and wealth evolution

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Wealth and knowledge are fundamental resources that evolve through exchanges and interactions between individuals and nations. In this work, we propose and analyze a nonlinear kinetic model that captures the bidirectional feedback between knowledge and wealth in a system of interacting agents. These agents interact in binary trades that account for international exchanges, with the possibility of agent transfers between different countries [1, 2].

Starting from a microscopic description of single-agent dynamics, where trade activity and personal knowledge levels influence each other, we derive the corresponding kinetic model. We then investigate its quasi-invariant asymptotic regime and equilibrium distributions. Furthermore, we derive the associated macroscopic equations governing population density, mean knowledge and national wealth, and we analyze how individual parameters shape equilibrium distributions for specific choices of microscopic coefficient functions.

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Wave propagation with powerless internal forces

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Nonlinear elastodynamics remains a challenging field, particularly when dispersive and dissipative effects must be considered to accurately model the mechanical response of real materials. While classical works such as [1, 2] provide a solid foundation, recent developments have introduced thermodynamically consistent models for hyperelastic materials with dispersive effects [3, 4, 5, 6]. In particular, the framework in [6] includes stress contributions from both a potential and *powerless internal forces*, which do not perform mechanical work and satisfy

$$\mathbf{T}_0 \cdot \mathbf{D} = 0,$$

where $\mathbf{T}_0$ is the powerless part of the Cauchy stress tensor and $\mathbf{D}$ the symmetric part of the velocity gradient.

In this work, we isolate the effect of powerless internal forces by neglecting the potential-derived stress. We show that, although they do not contribute to mechanical power, these forces induce nonlinear dispersion effects that influence wave dynamics, becoming relevant at large strain rates and enabling the propagation of traveling wave solutions.

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Thermal convection in bidisperse porous media

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The onset of thermal convection in a bidisperse porous medium saturated with an incompressible fluid, considering non-uniform internal heat generation and a constant temperature gradient induced by heating from below, is investigated. The macropore fluid dynamics is described by a Brinkman model, while a Darcy model governs the micropore motion, ensuring a more accurate representation of momentum transfer across different scales. The system is formulated under the Oberbeck-Boussinesq approximation, incorporating buoyancy effects through temperature-dependent density variations.

Both linear and nonlinear stability analyses are performed to assess the influence of key physical parameters on the stability of the basic state. The critical conditions for the onset of convection are determined by solving a generalized eigenvalue problem via the Chebyshev-τ method. Nonlinear stability is investigated within the energy theory framework using the differential constraints method. A golden section algorithm is implemented to compute and compare the critical thresholds from both analyses, providing insights into the proximity of linear and nonlinear stability bounds.

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Propagation of elastic waves in inhomogeneous media

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The theoretical foundation of a localization method for crack detection in a concrete sample is presented, based on the time of arrival of the elastic wave generated by the crack formation to a group of sensors positioned on the boundary of the sample.

The equations of motion for the elastic waves and a body force term which accounts for the sudden formation of a crack, which were introduced in a previous paper in the case of homogeneous media [1], are extended to the inhomogeneous case. Some numerical experiments are performed to assess the validity of the model and to make a comparison between inhomogeneous and homogenized media.

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New estimates for cross diffusion equations

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In a work in common with Helge Dietert, we propose the complete proof of an estimate in a Hölder space $C^{0,\alpha}$ (for some $\alpha > 0$) for solutions to parabolic equations of the form

$$a_0(t,x)\,\partial_t u(t,x) - \Delta u(t,x) = f(t,x), \qquad \text{on} \quad [0,+\infty[\times\Omega,$$

for f lying in L^p type spaces, and a_0 bounded above and below (by a strictly positive constant), coupled with homogeneous Neumann boundary condition

$$\nabla u(t,x)\cdot n(x) = 0, \qquad \text{on} \quad [0,+\infty[\times\partial\Omega,$$

in the special case when $\partial_t u(t,x) \geq 0$.

As a consequence of this estimate, we show new existence results of strong solutions for cross diffusion systems of the type

$$\partial_t m(t,x) - \Delta(m(t,x) + m(t,x)\,n(t,x)) = m(t,x)\,(1 - m(t,x)^{k_{11}} - n(t,x)^{k_{12}}),$$

$$\partial_t n(t,x) - \Delta n(t,x) = n(t,x)\,(1 - m(t,x)^{k_{21}} - n(t,x)^{k_{22}}),$$

in space dimension 3, and for well chosen $k_{ij} > 0$.

An attempt at energy equality for weak solutions to non-Newtonian viscous fluids in the shear thinning case

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In [1], we study the IBVP of the power-law fluids in a spatially periodic domain:

$$\begin{cases} v_t - \nabla \cdot ((\mu + |\mathcal{D}v|^2)^{\frac{p-2}{2}} \mathcal{D}v) + v \cdot \nabla v + \nabla \pi_v = 0, \\ \nabla \cdot v = 0, \text{ in } (0,T) \times \Omega, \\ v(0,x) = v_0(x), \text{ on } \{0\} \times \Omega, \end{cases} \tag{1}$$

where $\Omega := (0,L)^3$, $L \in (0,\infty)$, is a cube and we prescribe spatially periodic boundary conditions.

The goal is the construction of a weak solution satisfying an energy equality. Indeed, in the 2-dimensional case, it is known the existence of global strong solutions for $p > 1$ and these satisfy the energy equality.
In the 3-dimensional case, for $p \geq \frac{11}{5}$ it is well known the existence of global strong solution, so these satisfy the energy equality, but for $\frac{9}{5} < p < \frac{11}{5}$, global weak solutions to system (1) satisfy merely the energy *inequality*.

The approaches used, in the singular case, with $\mu > 0$, and $v_0 \in J^2(\Omega)$, allows to prove an energy equality which involves other quantities.

It is interesting to observe that the result is in complete agreement with what is known for the Navier-Stokes equations. Furthermore, in both cases, the additional dissipation, which measures the possible gap with the classical energy equality, is only expressed in terms of energy quantities. Moreover, from a physical view point the energy relation would add a dissipative quantity which is not justifiable. Thus, the question arises of investigating the nature of these additional dissipation terms: they could be due to turbulence phenomena or to the weak regularity properties of the solution.

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A unifying perspective on defect evolution in light of Configurational Mechanics

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In this contribution, we develop a theoretical study of defect propagation in solid media, in which a unifying approach to the onset and evolution of both topological and material defective phenomena is addressed. We recall that, in the field of Continuum Mechanics, we speak of "defect" to denote both alterations in the topology of a material—such as crack advancement or rotation and changes in the volume of inclusions or voids [1]—and inhomogeneities, i.e., spatial changes in the material properties of a medium. [2, 3].

The kinematic behavior of defects is specified by assigning their specific mode of propagation and, based on the principles of Eshelbian (Configurational) Mechanics [2, 3], we compute the energy release rate, or driving force, associated with their advancement. Furthermore, we recognise the individual contributions of this force, categorizing them into mechanical, topological, and those arising from material inhomogeneities [2, 3].

To better illustrate the application of our modelling framework, we review some classical examples of defect mechanics taken from the existing literature, integrating them into our approach [4]. Finally, we compare our findings with results obtained from variational methods grounded in conservation laws [1, 5].

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On well-balanced finite difference, finite volume and discontinuous Galerkin schemes for the Einstein-Euler system of general relativity

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In this talk we present a new family of well-balanced conservative CWENO finite difference schemes and discontinuous Galerkin (DG) finite element schemes with subcell finite volume (FV) limiter for the numerical solution of the Einstein–Euler equations of general relativity based on a first order hyperbolic reformulation of the Z4 formalism. The first order Einstein-Euler Z4 system, which is composed of 64 equations, is analysed and proven to be strongly hyperbolic for a general metric. The well-balancing is achieved for arbitrary but *a priori* known equilibria by subtracting a discrete version of the equilibrium solution from the discretized time-dependent PDE system. Special care has also been taken in the design of the numerical viscosity so that the well-balancing property is achieved. As for the treatment of low density matter, e.g. when simulating massive compact objects like neutron stars surrounded by vacuum, we have introduced a new filter in the conversion from the conserved to the primitive variables, preventing superluminal velocities when the density drops below a certain threshold, and being potentially also very useful for the numerical investigation of highly rarefied relativistic astrophysical flows.

We furthermore present a novel family of central WENO finite difference schemes for a new first order reformulation of the classical BSSNOK system.

Thanks to these improvements, all standard tests of numerical relativity are successfully reproduced, reaching four main achievements: (i) we are able to obtain stable long term simulations of stationary black holes, including Kerr black holes with extreme spin, which after an initial perturbation return perfectly back to the equilibrium solution up to machine precision; (ii) a (standard) TOV star under perturbation is evolved in pure vacuum ($\rho = p = 0$) up to $t = 1000$ with no need to introduce any artificial atmosphere around the star; and, (iii) we solve the head on collision of two punctures black holes, that was previously considered un–tractable within the FO-Z4 formalism, (iv) we perform a stable long-time evolution of a rotating binary black hole merger based on the new CWENO schemes for first order reformulation of the BSSNOK system.

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Bifurcations in Floquet theory

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Many dynamical systems possess a periodic solution whose stability is interesting. Typically, such solutions can be found in phase space of mechanical systems, or in fluid dynamical systems such as in the case of fully developed convective motion, or in extended space $\mathbb{R}^n_x \times \mathbb{S}^1_t$ of biological systems with coefficients periodically depending on time.

The behavior of periodical orbits is generally studied using the Poincaré map of first return to a section transversal to the flow. A linearization of this map gives crucial information on the stability of the orbit, and it is strictly connected to Floquet theory.

We can then construct bifurcation manifolds in the space of the parameters appearing in the system, as a pull-back from the space of invariants of the Floquet monodromy map, and we discuss a systematic procedure to determine, in parameter space, the marginal manifolds across which the periodic solution changes, in different ways, its local qualitative behavior.

We also apply the theory to a model system in which the time dependency of the coefficients is expressed in the lowest harmonics, to have a time periodic system which is significant but also depends on few parameters.

Hyperbolic effects in Michaelis-Menten uptake of dopamine in striatum

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An hyperbolic model is introduced in the context of Rational Extended Thermodynamics (RET) for the description of the dopamine diffusion in the striatum with the non-linear Michaelis-Menten uptake term. This model has the main features of hyperbolic system in addition it has the main characteristics of the parabolic Nicholson's model. The field equations are analytically integrated for some particular cases and the differences between the hyperbolic and the parabolic models are illustrated. Finally, some simulations were employed utilizing the Finite Element Method (FEM) to show the solutions of the model and highlight the differences from the parabolic one.

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Interpretable data modelling approach inspires a multiscale model for the thermo-hygro-mechanical behavior of spider silks

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Spider silks, renowned for their remarkable mechanical properties, may exhibits supercontraction—a significant fiber shrinkage in humid environments. The macromolecular mechanisms driving this phenomenon remain unclear, while understanding these processes is fundamental for advancing biomimetic materials with tunable properties in a suitable range of environmental conditions. In this work, we exploit interpretable machine learning techniques to uncover key relationships between silk protein composition and macroscale fiber behavior.

Specifically, we adopt a symbolic data modelling technique that generates explicit analytic formulas to fit data points using the evolutionary process of Genetic Programming (GP). It is the "Evolutionary Polynomial Regression" (EPR) method integrating regression capability and GP paradigm [1]. This approach allows for minimizing model complexity while effectively capturing the role of variables in multiscale modeling, in a classical Pareto front [2]. Our investigation leverages recent multiscale experimental data across various spider silk types [3], to systematically explore how the primary sequence of the main proteins composing the silk fiber influence the material behavior.

Novel data-driven insights are then integrated into a microstructure-inspired model to predict the thermo-hygro-mechanical behavior of spider silks. As a result, we obtain a fully physically based model for predicting the fiber behavior based on protein primary sequences properties [4].

The proposed model is validated by comparing experimental data with theoretical predictions obtained from both machine learning and the enhanced microstructure-inspired model. This work improves the understanding of spider silks behaviour by establishing a connection between the primary structure of silk proteins and the macroscopic response of the fibers, providing valuable insights for the design of biomimetic silk materials.

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On plasma densities maximizing laser wake field acceleration

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We report on the development [1] of the multi-step preliminary analytical procedure of [2] to tailor the initial density $\widetilde{n_0}(z)$ of an inhomogenous cold diluted collisionless plasma to a very short and intense plane-wave laser pulse travelling in the positive z direction, so as to control the formation of the plasma wave (PW), its wave-breaking (WB) at density inhomogeneities, the self-injection of low-charge bunches of plasma electrons in the PW by the first WB at the density down-ramp, and to maximize the initial stages of the laser wakefield acceleration of the latter. The results may help in simplifying the study of extreme acceleration mechanisms of electrons, which have many very important applications. The procedure partially *inverts* the determination of the motion of the plasma electrons given $\widetilde{n_0}$ and the laser pulse. Such a determination is based on a "post-hydrodynamic" (multi-stream) fully relativistic plane model (valid as long as the pulse deformation can be neglected), which up to WB reduces to an improved fully relativistic plane hydrodynamic model [3, 4, 5] whereby the pulse is modeled as a plane wave travelling in the z direction and the Lorentz-Maxwell and electrons' fluid continuity equations are reduced to a family (parametrized by $Z > 0$) of decoupled pairs of Hamilton equations; Z pinpoints the infinitesimal layer of electrons having coordinate $z = Z$ for $t \leq 0$, while $\xi = ct - z$ replaces time t as the independent variable. To make the *inversion formulae* maneagable, we stick to slowly varying density profiles $\widetilde{n_0}(z)$. We check the effectiveness of the $\widetilde{n_0}$ resulting from the inversion formulae, and can then further improve it by fine-tuning, solving again the direct problem (first the equations of our plane model, then those obtained with Particle-In-Cell codes).

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Effects of temperature, humidity and disorder in phase transformation phenomena of multi-stable systems

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Multi-stable behavior at the microscopic length-scale is fundamental for phase transformation phenomena observed in many materials. These phenomena can be driven not only by external mechanical forces but are also crucially influenced by disorder and thermal fluctuations. Disorder, arising from structural defects or fluctuations in external stimuli, disrupts the homogeneity of the material and can significantly alter the system's response, often leading to the suppression of cooperativity in the phase transition. Temperature can further introduce novel effects, modifying energy barriers and transition rates.

We study prototypical models able to represent the behavior of several biomacromolecules and phase transformation phenomena in solid mechanics. In the framework of Statistical Mechanics we can interpret the experimentally observed thermal effects with the theoretical force–extension relation characterized by a temperature-dependent force plateau (Maxwell stress) and a force peak (nucleation stress) [1]. Moreover, by using a mean field approach, we are able to include the effects of disorder in the framework of a replica symmetry theory [2]. Finally, we consider a predictive model for the hygro-thermo-mechanical behavior of complex materials such as spider silks, exhibiting an effect known as *supercontraction*, conceptualizing this phenomenon as a solid-solid phase transition driven by humidity [3]. As wetting increases, the system undergoes a transition, at the network scale, from a hard dry state to a soft amorphous wet state. We model these states by using a two-well free energy function dependent on molecular stretch, with transition energy modulated by humidity. We are thus able to deduce that supercontraction can be interpreted as a solid-solid phase transition.

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Emerging properties of the degree distribution in large non-growing networks

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The degree distribution is a key statistical indicator in network theory, often used to understand how information spreads across connected nodes. In this talk, we focus on non-growing networks formed through a rewiring algorithm and develop kinetic Boltzmann-type models to capture the emergence of degree distributions that characterize both preferential attachment networks and random networks.

Under a suitable mean-field scaling, these models reduce to a Fokker–Planck-type partial differential equation with an affine diffusion coefficient, that is consistent with a well-established master equation for discrete rewiring processes. We further analyze the convergence to equilibrium for this class of Fokker–Planck equations, demonstrating how different regimes—ranging from exponential to algebraic rates—depend on network parameters. Our results provide a unified framework for modeling degree distributions in non-growing networks and offer insights into the long-time behavior of such systems.

Viscous shock waves in relativistic fluid dynamics

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In the classical Navier-Stokes-Fourier description of non-relativistic dissipative fluid dynamics, prototypical shock waves are planar solutions with a heteroclinic profile. This picture, which goes back to Gilbarg (1951), has a straightforward extension to the well-known descriptions of relativistic dissipative fluid dynamics proposed by Eckart (1940) and Landau (1953). However, the Eckart and Landau theories are of limited use since they violate the causality principle, and alternative theories have been developed that are causal; of these, Rational Extended Thermodynamics by Ruggeri et al. (since 1983) is probably the most appropriate one currently available.

This talk considers a class of recently proposed formulations which achieve causality by representing viscosity and heat conduction via second-order hyperbolic operators in the standard fluid dynamical variables. We show that only some of these formulations admit regular profiles for shock waves of arbitrary amplitude.

A multiscale approach to model bacterial ecology and sorption interaction in marine sediment

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A macroscopic model for bacterial colonies (i.e. biofilm) growth and evolution in a homogeneous marine porous medium (i.e. marine sediment) is derived by upscaling a one-dimensional bacterial-scale multispecies biofilm model. The flow through the porous medium is assumed in a laminar and convection-dominated regime. The formal multiscale asymptotic method is applied to the mesoscale coupled system of elliptic-hyperbolic equations describing biofilm growth and evolution. The proposed model hypothesizes a dual-species biofilm uptaking the same substrate. The loss of biomass due to the shear forces i.e. detachment process, and the attachment of bacteria clusters in quiet zones of the domain, i.e. attachment process, are modelled. Moreover, the model considers the ecology of the suspended biomass and their interaction (through attachment and detachment) with the biofilm. Finally, heavy methal interaction with biofilm is taking into account by modelling sorpion process and metal interactions with bacteria metabolism. The upscaling procedures end up with a stiff system of hyperbolic equations that are solved numerically. The results prove model consistency and different simulation scenarios have been investigated by varying the following parameters: attachment velocity, detachment coefficient, and fluid flow rate. The mixed-culture biofilm assumption was found to significantly affect the overall system performance, and the model outputs qualitatively agree with the physical expectations.

Subharmonic instabilities and the emergence of oscillatory patterns in reaction-diffusion systems with nonlinear diffusion

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The classical mechanisms for the onset of oscillations in reaction-diffusion systems include Hopf instability, wave instability, or the presence of periodic forcing. More recently, it has been observed that nonlinear diffusion, both chemotactic [1] and cross-diffusion [2], can induce oscillations even in the absence of any Hopf (or wave) bifurcation or external forcing.

The emergence of these oscillatory patterns is driven by a secondary bifurcation: the stationary pattern, initially formed through a Turing bifurcation, loses stability due to the destabilization of subharmonics of the fundamental wavenumber k_c of the stationary Turing pattern. Far from equilibrium, this instability triggers an efficient energy transfer to its next subharmonics ($k_c/2$ and/or $3k_c/2$).

To describe the emergence of this secondary bifurcation, we shall derive the normal form for the unstable amplitudes. Using a center-unstable manifold expansion, the stable modes are projected on the invariant manifold generated by the unstable modes that satisfy the resonance conditions [3]. The analysis of the resulting amplitude system allows us to examine the stability of the saturated fundamental mode against disturbances with wavenumbers $k_c/2$ and/or $3k_c/2$, and to predict the critical amplitude threshold at which subharmonic excitation sets in.

Finally, we will show that a further increase in the bifurcation parameter leads to a cascade of period-doubling bifurcations, ultimately giving rise to spatio-temporal chaos.

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Spatial feedbacks and self-organization in Posidonia oceanica meadows under environmental stress

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Feedback mechanisms that vary at different length scales can give rise to striking large-scale spatial patterns in ecosystems [1]. In this work, we analyze a model describing the dynamics of Posidonia oceanica [2], a seagrass species that supports high biodiversity and is known to be affected by long-range competitive processes alongside short-range facilitative effects. The model exhibits two homogeneous equilibrium states: a continuous seagrass meadow and a bare seabed,which may coexist under specific parameter regimes. Our analysis reveals that the interplay of scale-dependent interactions plays a fundamental role in driving spatial pattern formation, with long-range competition serving as a key mechanism. Through the analysis of Turing bifurcation diagrams, we show that these patterns emerge as a potential adaptive response of the ecosystem to moderate environmental stress, allowing the system to reorganize and maintain biomass under suboptimal conditions. However, when environmental stress becomes too intense, the system may fail to transition to a patterned state and instead cross a critical tipping point, ultimately leading to the collapse of the meadow.

This work is part of the activities of the PRIN-PNRR 2022 project "Mathematical Modeling of Biodiversity in the Mediterranean sea: from bacteria to predators, from meadows to currents" (Project code P202254HT8 - CUP B53D23027760001)

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Bifurcations in symplectic Floquet flows

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Equilibria and periodic solutions play a pivotal role in understanding of the evolution of a dynamical systems [1]. In particular, the spectral type of the linearization near a given periodic orbit affect the structure of neighboring orbits and also that of (un)stable manifolds of the orbit. Periodic solutions are ubiquitous: they easily appear in mechanical systems, and they also appear in time-dependent systems subjected to periodic forcing [2].

Most systems depend on parameters, periodic orbits develop along with the parameters, and their spectral type changes as well. We present a systematic methodology to identify the loci in parameter space across which the spectral type of a periodic solution undergoes qualitative changes (called *marginal manifolds*). These loci arise as the pull-back to physical parameter space of algebraic conditions imposed on the space of invariants of the Floquet map associated with the system [3].

Key to this investigation are the inherent symmetries of the characteristic polynomial of the Floquet matrix which, due to the symplecticity of the map, is bound to be palindromic [4]. This condition forces the eigenvalues to form, under generic conditions, quartuplets whose structure can qualitatively change only under special degenerate conditions [5]. By formulating semi-algebraic equations in the coefficients of the characteristic polynomial and subsequently mapping these to the space of physical parameters, we delineate the boundaries of spectral stability transitions.

An immediate application of the theoretical framework we developed can be done to the classical *elliptic restricted three-body problem*. Its Lagrange periodic orbits [6] can be investigated to illustrate how the interplay between Floquet theory, algebraic geometry, and bifurcation theory reveals critical thresholds in dynamical systems, offering insights into bifurcations and stability loss mechanisms.

This work is a natural continuation of that done by the author on linearization of equilibria in generic and mechanical systems [7, 8].

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Eckhaus instability in the Sutton problem: dynamics of two competing modes

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The Sutton problem [1] describes the thermoconvective instability which is induced by heating a uniform porous layer from below, but it differs from the classical Darcy-Bénard problem by allowing for a constant fluid suction through the upper and lower boundaries. The throughflow velocity is modelled by the Péclet number (Pe), while the strength of buoyancy forces by the Rayleigh number (Ra). A resonance phenomenon has been detected by the weakly nonlinear analysis performed in [2], as forcing term at second order has a component along a fundamental mode of the linear operator. This is consequence of the condition $\mathrm{Ra(k)} = \mathrm{Ra(2k)}$, which leads to the simultaneous generation of two modes that interact at the onset of instability [3].

The aim of this talk is to refine the weakly nonlinear theory to investigate the local bifurcation dynamics of two competing modes at the onset. The analysis relies on the key assumption of a weak throuhgflow modelled by $\mathrm{Pe} \ll 1$, allowing the Péclet number to be employed as small expansion parameter. The weakly unstable regime is then achieved by perturbing the critical Rayleigh number by $O(\mathrm{Pe}^2)$. This choice allows us to determine analytically a pair of coupled amplitude equations, whose steady solutions are then investigated numerically by means of a continuation method. In a first instance, the analysis is conducted at a specific wavenumber, while afterwards, modulation over a slow lengthscale is introduced and the study of Eckhaus instability is presented by tuning the deviation κ characterising the modulation [4]. The resulting bifurcation diagrams reveal a rich dynamics with up to seven equilibria, which are analysed and discussed.

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Numerical Analysis of the von Neumann Equation in the Semiclassical Regime

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In quantum mechanics, the wave function $\psi(t,X) \in \mathbf{C}$ of a point particle of mass m subject to the force field deriving from the potential $V \equiv V(X) \in \mathbf{R}$ satisfies the Schrödinger equation

$$i\hbar\partial_t\psi(t,X) = -\frac{\hbar^2}{2m}\Delta_X\psi(t,X) + V(X)\psi(t,X)\,, \qquad \psi\big|_{t=0} = \psi^{in}\,.$$

The quantum Hamiltonian $\mathcal{H} := -\frac{\hbar^2}{2m}\Delta_X + V(X)$ is self-adjoint on L^2 for a suitable class of potentials V and the wave function $\psi(t,X)$ is given in terms of its initial value ψ^{in} by $\psi(t,X) = \exp(-\frac{it\mathcal{H}}{\hbar})\psi^{in}(X)$. Mixed quantum states involve statistical mixtures with weights $\lambda_0, \lambda_1, \ldots, \lambda_n, \ldots \geq 0$ of L^2-orthogonal systems of wave functions, of the form

$$R(t,X,Y) = \sum_{n\geq 0} \lambda_n \exp(-\tfrac{it\mathcal{H}}{\hbar})\psi^{in}(X)\overline{\exp(-\tfrac{it\mathcal{H}}{\hbar})\psi^{in}(Y)}\,, \qquad \text{where } \textstyle\sum_{n\geq 0}\lambda_n = 1\,.$$

The function $R(t,X,Y)$ is referred to as the "density matrix" and satisfies the von Neumann (or quantum Liouville) equation

$$i\hbar\partial_t R(t,X,Y) = -\frac{\hbar^2}{2m}(\Delta_X - \Delta_Y)R(t,X,Y) + (V(X) - V(Y))R(t,X,Y)\,.$$

In the semiclassical regime, the Planck constant $\hbar \ll 1$ the typical action of the particle, and computing numerically the propagator $\exp(-\frac{it\mathcal{H}}{\hbar})$ is a stiff problem which typically involves time steps Δt and spatial mesh size of order $\hbar$ or smaller. It was noticed already in [1] that time-splitting methods with time steps much larger than $\hbar$ lead to a good evaluation of observables (such as the mass, momentum, and energy densities), with uniform in $\hbar$ error estimates [3]. However, these works leave aside the spatial discretization issue. Clever variants of the Gaussian wave packet method [4] help reducing the stiffness of the problem, but still require spatial mesh size of order $\sqrt{\hbar}$.

In this talk, we present a new approach [2] to the numerical analysis of the von Neumann equation which removes all stiffness as $\hbar \ll 1$. We discuss its validity and limitations, and illustrate it with some numerical test cases.

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Nonlinear brain dynamics: spikes, waves, fronts, and dementia

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It is well established that various dynamical processes, including transport phenomena and wave propagation, occur within the brain. These processes are driven by mechanisms grounded in diverse physical principles, such as action potentials traveling along axons, protein transport within intracellular and extracellular spaces via diffusion, electrophysiological depolarization waves observed during cortical spreading depression, and cellular migration in cancerous tissues. Despite their differences, these phenomena share common constraints imposed by tissue heterogeneity, intricate cortical geometry, and the complex topology of the human brain connectome. Notably, it has been hypothesized that neurodegenerative diseases are associated with the transport and subsequent amplification of toxic proteins along axonal pathways. We will demonstrate that, beginning from an initial seeding region, these proteins propagate outward in the form of traveling fronts across the brain, progressively damaging neuronal tissues and consequently disrupting the brain's natural oscillatory dynamics. By coupling protein transport with neuronal oscillatory processes, we aim to gain novel insights into the progression of cognitive dysfunction in dementia, achieved through the study of nonlinear waves emerging from neuronal mass models defined on dynamically evolving networks.

Boundary conditions at material surfaces for dispersive fluids

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We focus on two classes of dispersive equations that are Euler-Lagrange equations for Hamilton action expressed in space-time with a Lagrangian depending not only on the thermodynamic variables but also on their spatial and temporal first derivatives. This is the case for the fluid model with capillarity (the Lagrangian depends on the density gradient) and for the fluid model with gas bubbles (the Lagrangian depends on the material derivative of the density); equivalent models also appear in the long-wave theory of free-surface flows.

We obtain a system of equations governing dispersive systems that are reversible in time; these include conservation laws for mass, momentum and energy, but also new boundary conditions with imposed motion (such as the piston problem) and boundary conditions for surfaces with energy deduced from molecular models between fluids and walls. Joint work with Sergey Gavrilyuk.

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Toxicity-driven motion of vegetation patterns

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During the last decades, extensive research has been focusing on the formation and stability of vegetation patterns in the context of dryland ecology. These studies have have highlighted how fluctuations in environmental parameters can produce severe consequences, such as desertification, and have been instrumental in pinpointing ecological indicators of climate change and ecosystem resilience [1].

A pivotal mechanism driving these dynamics is the positive feedback between local biomass and water. However, vegetation patterns are also observed in non-water-limited ecosystems, suggesting the presence of alternative mechanisms, such as the presence of toxic compounds, that may be crucial in shaping their formation, evolution and stability [2]. The negative feedback associated with plant toxicity is evidenced by field observations showing that the same plant species cannot thrive in the same area over time due to the accumulation of soilborne pathogens and autotoxic compounds [3].

The main purpose of this presentation is to elucidate the role played by toxicity on the dynamics of vegetation patterns emerging in flat semi-arid regions. To this aim, an extension of the Klausmeier model [4] accounting for the presence of toxic compounds [5] has been considered. In detail, linear stability analysis has been employed to identify the parameter regions in which patterns may arise as well as to elucidate the primary features of patterned solutions at the bifurcation threshold. Subsequently, multiple-scale weakly nonlinear analysis has been used to derive the equation governing the evolution of pattern amplitude close to the onset. Numerical simulations have been also conducted to validate the analytical predictions and to provide further ecological insights into this complex phenomenon.

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Manifold connections and the transport of small bodies through mean motion resonances in the Solar system

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The talk will focus on a study of how the transport of small bodies through the orbit of Jupiter in the Solar System is governed by the heteroclinic intersections between the stable and unstable manifolds of the unstable periodic orbits corresponding to each one of the main mean motion resonances between the body's and Jupiter's orbits. These manifolds have been extensively discussed in literature in the case of the co-orbital resonance (manifolds of the periodic orbits around the collinear Lagrangian points), but to a lesser extent for other important mean motion resonances. We explicitly compute manifolds of the orbits of other mean motion resonances and show their correspondence with the hyperbolic structures found numerically (through FLI maps) in the circular and also in the elliptic RTBP. Direct heteroclinic connections through the stable and unstable invariant manifolds of various families, internal or external to the orbit of Jupiter, will be discussed.Work in collaboration with C. Efthymiopoulos (UniPD).

From microscopic delayed interactions to nonlocal diffusion in traffic flow

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Traffic dynamics are strongly influenced by drivers' perceptual and cognitive mechanisms, such as their ability to anticipate upstream flow conditions and the latency in their response to perceived changes. In congested scenarios, these effects become especially relevant: drivers do not react instantaneously to local density, but instead make decisions based on a forward-looking perception, with an unavoidable reaction delay. To model these aspects coherently, we start from a microscopic follow-the-leader model with weighted nonlocal interaction and an explicit delay in the individual dynamics. Through an asymptotic expansion in the delay parameter, we derive a macroscopic equation in which diffusion emerges naturally as a correction induced by the reaction time.

In the present talk, I illustrate the key mechanisms behind the derivation, emphasizing how the delay acts as a source of intrinsic diffusion, and how the anticipation horizon influences the qualitative behavior of the system. I also present numerical simulations that illustrate the regularizing effect of delay-induced diffusion and the influence of the anticipation horizon on the evolution of traffic density. These results show how the combined effect of delay and nonlocality can promote the emergence of smoother, more stable traffic trajectories, even in highly congested regimes. This provides a physically meaningful way to capture complex traffic dynamics under realistic driving behavior.

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Travelling pulses of vegetation driven by autotoxicity effects

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In this talk, an extension of the 1D Klausmeier model that accounts for the toxicity compounds is considered and the occurrence of travelling stripes is investigated as in [1]. Numerical simulations are firstly conducted to capture the qualitative behaviours of the pulse–type solutions and, then, geometric singular perturbation theory is used to prove the existence of such travelling pulses by constructing the corresponding homoclinic orbits in the associated 4-dimensional system. A scaling analysis on the investigated model is performed to identify the asymptotic scaling regime in which travelling pulses can be constructed.

Biological observations are extracted from the analytical results and the role of autotoxicity in travelling patterns is emphasized. Finally, the analytical solutions are compared with the numerical ones, leading to a good agreement that supports the stability of the constructed profiles. Numerical investigations are also carried out in order to gain additional information on vegetation dynamics.

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Exploring pattern formation in soil organic dynamics: insights for circular economy

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Pattern formation in diffusive-chemotaxis models has become increasingly important for understanding spatial structures in biological, ecological and chemical systems. In soil ecosystems, specific bacteria involved in soil organic carbon degradation exhibit motility and chemotactic behaviour, as shown in both experimental and field studies. To capture the formation of hotspot soil aggregations caused by the spatial arrangement of bacteria and microorganisms, a reaction-diffusion-chemotaxis model (MOMOS) is used. In this talk, we focus on the self-organization properties of the MOMOS model for soil carbon dynamics and explore pattern formation beyond the conventional framework of diffusive-chemotaxis-driven instabilities by analysing parameter space regions associated with reactive equilibria. Exploring spatial pattern formation in the MOMOS model is relevant within the context of the Circular Economy. Gaining a deeper understanding of how carbon behaves and is distributed in soil over time can in fact support regenerative agricultural practices, improve organic waste management and optimize farming methods.

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Singularity for Euler, Navier-Stokes, and Boltzmann equations

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For the study of gas motions, there are three main models: the Euler equations, Navier–Stokes equations, and Boltzmann equation. These models have distinct features within singular layers. The aim of the present article is to study their different behaviors within the initial layer. We illustrate this through explicit calculations for propagation of discontinuities and formation of nonlinear waves in the solutions of the Riemann problem.

Exploring immune pathways and remyelination in Multiple Sclerosis

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In this talk, I shall review a class of recently developed mathematical models aimed at capturing the primary mechanisms that drive the initial stages of Multiple Sclerosis (MS). A fundamental debate persists in the immunological community concerning the triggers of the immune cascade that initiate MS pathology. The first scenario involves activation of local microglia, recruitment of systemic immune responses, and oligodendrocyte apoptosis [1, 2, 3, 5, 6], whereas the second scenario investigates cytokine-mediated modulation of macrophage activation [7]. We proposed the following reaction-diffusion-chemotaxis model for $m(t,x)$, the density of activated macrophages, $c(t,x)$ the density of cytokines, and $d(t,x)$ the density of oligodendrocytes [4]:

$$\begin{cases} \partial_t m = \Delta m - \nabla \cdot (\chi(m)\nabla c) + \frac{c}{\xi+c}\, m(1-m), & (x,t) \in \Omega_T \\ \partial_t c = \frac{1}{\tau}\left(\epsilon \Delta c + \delta d - c + \beta m\right), & (x,t) \in \Omega_T \\ \partial_t d = rF(m)\, m\,(1-d)\,, & (x,t) \in \Omega_T \end{cases} \tag{2}$$

with $\chi(m) = \chi\frac{m}{1+m}$, $F(m) = \frac{m}{1+m}$, $\Omega_T = (0,T) \times \Omega$, where Ω is a bounded domain in $\mathbb{R}^n$ $(n = 1, 2)$ with smooth boundary $\partial\Omega$ on which no-flux boundary conditions are imposed. Upon varying the parameter ξ that rules the effect of cytokines on the macrophage activation rate, the model describes the two different scenarios.

Employing both analytical and numerical methods, we demonstrate that the model supports the emergence of various demyelination patterns. Utilizing biologically realistic parameters from existing literature, we show that the asymptotic solutions of the model system (2) replicate pathological features closely matching MRI findings.

Bifurcation analysis reveals substantial differences in pathological outcomes between the two activation scenarios. The "innate immunity" scenario produces highly aggressive pathology, characterized by intense focal inflammation and rapid progression. In contrast, the "cytokine-mediated" scenario results in less aggressive pathology, exhibiting lower inflammation levels and significantly slower disease progression.

In the last part of the talk, we will discuss ongoing extensions of the model that integrate endogenous regenerative processes driven by acute inflammation stages, aimed at promoting remyelination—a regenerative effort robustly documented during early disease phases but frequently impaired in chronic MS lesions.

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A Hamilton-Jacobi approach to nonlocal kinetic equations

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Highly concentrated patterns have been observed in a spatially heterogeneous, nonlocal, kinetic model with BGK type operators implementing a velocity-jump process for cell migration directed by the nonlocal sensing of either an external signal or the cell population density itself. We describe, in an asymptotic regime, the precise profile of these concentrations which, at the macroscale, are Dirac masses. Because Dirac concentrations look like Gaussian potentials, we use the Hopf-Cole transform to calculate the potential adapted to the problem. This potential, as in other similar situations, is obtained through the viscosity solutions of a Hamilton-Jacobi equation. In the linear case, when the heterogeneous external signal is given, we show that the concentration profile obtained after the diffusion approximation is not correct and is a simple eikonal approximation of the true H-J equation. In the nonlinear case, when the signal occurs from the cell density itself, it is shown that the already observed linear instability (pattern formation) occurs when the Hamiltonian is convex-concave. Conditions for the nonlinear instability are found.

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Mixed Boltzmann-BGK and fluid-kinetic model for inert gas mixtures

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We present recent analytical results concerning two different models for inert gas mixtures. The first is a kinetic mixed Boltzmann-BGK model for mixtures of monatomic gases, where each binary collision can be chosen to be described either by bi-species Boltzmann operators or by binary BGK terms; the second is a fluid-kinetic model for binary mixtures, in which one of the species is much heavier than the other, derived from the mixed model in which intra-species collisions are modeled by Boltzmann operators and inter-species by BGK ones. We prove the consistency properties and derive the explicit form of the Euler hydrodynamic equations for both models.

For the mixed one, we then prove that the first-order corrections of the macroscopic fields needed to close the Navier-Stokes equations can be explicitly determined for the Maxwell molecule interaction potential.

For the fluid-kinetic description, via a suitable micro-macro decomposition we obtain for each species a coupled system composed of a microscopic equation, in which the unknown is the the kinetic correction to Maxwellian distribution, and balance equations, in which the unknowns are the macroscopic fields of the species. We then prove the equivalence of the coupled system with the Boltzmann-BGK formulation. Afterwards, we compute the decay rate of the species velocities and temperatures and the convergence rate of the distribution function to its relative Maxwellian species distribution.

Shock wave problem for Navier-Stokes equations for mixtures derived from a mixed Boltzmann-BGK model

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In classical kinetic theory the dynamics of inert gas mixtures is naturally described through integro-differential Boltzmann equations for species distribution functions [1]. However, the analytical and numerical study of these equations is rather difficult due to the complexity of the nonlinear integral Boltzmann operator that describes the collision mechanisms in detail. For this reason, starting from the BGK model for a single gas proposed by Bhatnagar, Gross and Krook [2] in 1954, several simpler models have been introduced.

In this talk, we present a mixed kinetic model for an inert gas mixture of monatomic gases that combines the positive aspects of both Boltzmann and BGK formulations [3]. In particular, it preserves the collision detail of the Boltzmann model in the dominant process, which is assumed to be the collisions between molecules of the same component of the mixture, and it takes advantage of the computational manageability of the BGK description in the remaining collisional phenomena. This mixed kinetic description is suitable to model the dynamics of gas mixtures in which the constituents have disparate masses, as it occurs in plasma physics [4] and in ε-mixtures of gases [5].

In the regime dominated by intra-species collisions, multi-velocity and multi-temperature hydrodynamic equations are derived with a classical Chapman-Enskog expansion technique up to Navier-Stokes order. The resulting equations constitute a system of balance laws for the main macroscopic fields of each gas, with proper source terms taking into account the inter-species interactions. We then investigate the structure of the stationary shock wave solution for the one-dimensional system of multi-temperature and multi-velocity Navier-Stokes equations for a binary mixture of noble gases for varying Mach number, and propose a comparison with a single-velocity formulation. In particular, we investigate the "shock thickness", i.e.. the size of the narrow region where the transition from an equilibrium state to another occurs, by varying the Mach number and for different mixture components.

This work is in collaboration with M. Bisi, M. Groppi (University of Parma) and G. Martalò (University of Pavia).

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Elastic Plateau–Rayleigh instability in soft cylinders: Surface elasticity and periodic beading

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The Plateau–Rayleigh instability shows that a cylindrical fluid flow can be destabilized by surface tension. Similarly, capillary forces can make an elastic cylinder unstable when the elastocapillary length is comparable to the cylinder's radius. While existing models predict a single isolated bulge as the result of an instability, experiments reveal a periodic sequence of bulges spaced out by thinned regions, a phenomenon known as beading instability. Most models assume that surface tension is independent of the deformation of the solid, neglecting variations due to surface stretch.

In this work, we assume that surface tension arises from the deformation of material particles near the free surface, treating it as a pre-stretched elastic surface surrounding the body. Using the theoretical framework proposed by Gurtin and Murdoch, we show that a cylindrical solid can undergo a mechanical instability with a finite critical wavelength if the body is sufficiently soft or axially stretched. Post-buckling numerical simulations reveal a morphology in qualitative agreement with experimental observations. Period-halving secondary bifurcations are also observed. The results of this research have broad implications for soft materials, biomechanics, and microfabrication applications where surface tension plays a crucial role.

Numerical inversion of Laplace transform via steepest descent path for transient waves in linear viscoelasticity

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The theory on transient waves in linear viscoelasticity is dealt in many papers and books, of which we refer for more details to the recent treatise [1].

Referring to uni-axial waves in a semi-infinite medium ($x \geq 0$), initially quiescent and denoting by r the "response" of the system chosen among the field variables, the pulse $r(x,t)$ can easily be expressed at any time $t > 0$ and position $x > 0$ by the Laplace transform. In fact we get, for media admitting a finite wave-front velocity c

$$r(x,t) = \frac{1}{2\pi i} \int_{Br} ds\, \widehat{r}_0(s)\, \mathrm{e}^{s[t-n(s)x/c]}\,,$$

in which s is the complex Laplace parameter and Br denotes the Bromwich path, *i.e.* a vertical line lying to the right of all singularities of $\widehat{r}_0(s)$ and $n(s)$. Furthermore the function $\widehat{r}_0(s)$ is the Laplace transform of the source pulse at $x = 0$ for $t \geq 0$, *i.e.* $r(0,t) = r_0(t)$ and $n(s)$ provides, for $s = \pm i\omega$, the complex index of refraction that can be found from the dispersion relation for the given medium.

In linear viscoelasticity the wave front velocity and complex index of refraction are obtained by one of the two material functions $J(t)$ (the creep compliance) or $G(t)$ (the relaxation modulus) of the viscoelastic model under consideration. For the simple Maxwell model the material functions read $J(t) = J_0\, t/\tau\, \Theta(t)\,,\ G(t) = G_0\, \mathrm{e}^{-t/\tau}\, \Theta(t)\,$, where $J_0\, G_0 = 1$, τ denotes the relaxation time and $\Theta(t)$ the unit step Heaviside function. In this case we get $c = 1$ and the complex index of refraction reads

$$n(s) = \sqrt{1 + \frac{1}{s\tau}}\,.$$

Although the Laplace transform method provides a formal solution to any signaling problem here considered through the Bromwich formula (1), it usually cannot provides exact analytic solution. When the integration can analytically be performed, it often leads to mathematical cumbersome solution even for the simplest forms of $n(s)$ because the presence of essential singularities and branch cuts.

In order to decrease the numerical difficulties for the Laplace inversion we propose to deform the original path of integration (the Bromwich path) into another equivalent to it (unless possible contributions of singularities) that turns out to be more convenient. Such path is the steepest descent path through the saddle points of the complex function

$$F_\mu(s) = s[1 - \mu\, n(s)]\,, \quad \mu = \frac{x}{ct}\ (0 \leq \mu \leq 1)\,.$$

We have the possibility to check the validity of our proposed method by comparing our numerical results with an analytic solution available n a particular case of a signaling problem for the Maxwell model, the so-called Lee-Kanter problem, discussed in [1]. Indeed in this case we have a solution in terms of a damped modified Bessel function of order 0. The above presentation is in a preliminary stage, but being promised other results will follow.

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Wave solutions for hyperbolic systems

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Determining exact solutions of hyperbolic systems is of great interest for studying nonlinear wave problems. Among others, the method of differential constraints provides an approach which, in principle, permits to characterize wave solutions for hyperbolic systems. To this end one is led to study a set of overdetermined nonlinear differential compatibility conditions which, in general, is very difficult to solve. Within such a theoretical framework, here, making use of the k-Riemann invariants, we propose a different strategy which, in some cases, allows to solve such a nonlinear compatibility conditions. The resulting exact solutions generalize the simple waves so that they can be useful for determining rarefaction waves also for non homogeneous hyperbolic systems. Examples concerning a ideal fluid are given.

A hyperbolic free boundary problem modeling biofilm growth

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We consider the radially symmetric case of a free boundary problem modeling the growth of multispecies biofilms, microbial agglomerates which proliferate in any natural and artificial environments. The biofilm is constituted by an arbitrary number of microbial populations, including dead cells and inert material. The densities of these populations satisfy a system of nonlinear first order hyperbolic equations, with the biofilm surface as a free boundary. The nutrients and by-products concentration satisfies a diffusion equation, and the free boundary $r = R(t)$ satisfies an integro-differential equation. We establish the existence and uniqueness of a local and then global solution, for any given initial data.

Lotka-Volterra-type kinetic equations for interacting species

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In this work, a kinetic framework to describe the time evolution of the size distribution densities of two interacting populations through Lotka-Volterra dynamics is proposed. The model is based on Boltzmann-type equations, where the evolution is driven by elementary binary interactions between individuals of the two populations. A distinctive feature of the model is the inclusion of a linear redistribution operator, inspired by those used in wealth-redistribution models, which captures the birth processes in both populations. We show that, under a suitable scaling limit, the kinetic description converges to a system of coupled Fokker-Planck equations. These equations govern the evolution of the distribution densities. At the macroscopic level, the behavior of populations is thus modeled by a Lotka–Volterra system of ordinary differential equations, with parameters explicitly derived from the microscopic interaction rules. The local equilibria are discussed and the relaxation of solutions towards these kinetic equilibria through the dynamics of their moments is studied, both analytically and numerically. The results highlight the multiscale connection between kinetic theory and classical population dynamics, offering a unified description of predator-prey systems.

This is a joint work with Andrea Bondesan, Giuseppe Toscani, Mattia Zanella.

Monotonic nonlinear stability of plane shear flows of Navier-Stokes-Voigt fluid in hydrodynamics and in MHD

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Monotonic nonlinear H^1-energy stability of laminar flows in a horizontal layer between two parallel planes filled with a Navier-Stokes-Voigt fluid (a non-Newtonian viscoelastic fluid), [1, 2, 3, 4], is studied. It is proved that the critical Reynolds numbers for monotone H^1-energy stability for the Couette and Poiseuille flows of the zero-order Navier-Stokes-Voigt fluid are the same as those found by Orr for Newtonian fluids [5]. However, the exponential decay coefficient depends on the Kelvin-Voigt parameter Λ [6]. Furthermore, a Squire theorem holds in the nonlinear case: the least stabilizing perturbations in H^1-energy are the two-dimensional spanwise perturbations [7, 8].

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Continuous limit of a traction-unstable metastructure

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The continuous limit for a metamaterial composed of several unit cells which are unstable under traction [1] is developed within the realm of elastodynamics. The resulting variational structure is crucially supplemented by a unilateral constraint, which turns the otherwise linear problem into a complementarity problem. Indeed, the absence of the unilateral constraint significantly alters the response of the system, notably missing the fundamental equilibrium state. We show that proper dealing with the unilaterally constrained Lagrangian [2, 3] leads to the reproduction at the macro-level of all the features exhibited by the microstructure. The Hamiltonian of the system is also discussed and plays a fundamental role in addressing the non-smooth transition between the equilibrium states.

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Modelling behavioural changes and vaccination in the transmission of respiratory viruses with co-infection

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Co-infection of two respiratory viruses occurs when an individual's respiratory system is simultaneously infected by two genetically different viral diseases. In the recent literature, several models can be found that provide useful insights into the co-infection dynamics [3, 4]. However, a crucial aspect remains neglected: the influence of information-driven changes in human behaviour. Indeed, it is well known that human behaviour plays a pivotal role in determining the course of an epidemic and the effectiveness of any containment measures. We propose a behavioural co-infection compartmental model to investigate the effects of the behavioural changes induced by the information about the epidemic status [1]. First, we consider the case where the containment measures are purely non-pharmaceutical and model the contact rate as a decreasing function of the information index, defined as a distributed delay that quantifies the level of information and rumours about the disease status [2]. We perform a qualitative analysis of the model through stability and bifurcation theory, in order to analyse the existence and stability of both endemic and co–endemic equilibria. Second, we extend the model to incorporate vaccination. The vaccination rates are assumed to increase with information about the prevalence of the diseases, and the contact rate to increase with the number of vaccinated individuals. Three information indexes are employed to quantify the information about the disease prevalence and vaccinated individuals. Among the main results, we show that behavioural changes may have a stabilising effect when only non-pharmaceutical measures are considered. In this case, sustained oscillations may turn into damped oscillations converging towards a steady state in which co-infection is endemic. Conversely, when both vaccination and non-pharmaceutical measures are considered, the effect of behavioural changes in contact patterns may have a destabilising effect.

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Two temperatures NSF theories for internal molecular relaxation processes as a limit of RET

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The paper aims to derive the Navier-Stoke-Fourier type equations with two temperatures as a coarse-grained theory of the rational extended thermodynamics for the energy exchange processes between molecular translational, rotational, and vibrational modes. The deduced equations can be applied to the three processes, that is, the process in which (i) the vibrational mode relaxes slowly, (ii) the rotational mode relaxes slowly and (iii) rotational and vibrational modes relax slowly as a unit. In case (i), the system of the field equations corresponds to the two temperature models derived in [1] and in case (iii), the system corresponds to the one derived in [2].

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Weakly non-linear Rayleigh waves within non-local elasticity

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Non-local elasticity has gained much attention in the literature for its ability to incorporate small-scale effects in the material behaviour, especially for micro- and nano-structures, whose dimensions are comparable with that of the underlying microstructure. However, the classical integral formulation of non-local elasticity, as presented by Eringen [1], often leads to ill-posedness, in light of the appearance of (possibly conflicting) extra boundary conditions, named constitutive. While recent developments have established a well-posed modification of non-local elasticity within the linear regime [2, 3, 4], its extension to the nonlinear framework remains largely unexplored. In this work, we address this gap by formulating a weakly nonlinear boundary value problem for Rayleigh waves propagating in an elastic half-space, that combines the well-posed non-local elasticity model with a third-order nonlinear expansion of the strain energy. The inclusion of high-order nonlinear terms provides an interesting insight concerning the material behaviour under large deformations, while addressing essential nonlinear dispersion effects. A closed form solution is arrived at by standard asymptotic methods. This study demonstrates that the well-posed non-local elasticity theory may be successfully extended to the nonlinear framework, and it provides an original viewpoint for modelling surface wave propagation, while overcoming the limitations of both the purely local as well as of the ill-posed non-local theory.

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Supercontraction-induced torsion in spider silks is just a dual Poynting effect

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Supercontraction represents an important counterintuitive shrinking behavior of silks wires under increasing humidity. While fibers shortening is due to the entropic recoiling of structural molecules in the wire direction induced by H-bonds disruption by water molecules, surprisingly fibers also exhibit a significant coupled torsion. Previous studies described this effect based on the unverified assumption of helical structural fibers in spider silks wires undergoing shrinking. Here, based on a recent theoretical discussion demonstrating the possible insurgence of torsion in compressed non linear fiber reinforced elastic cylinders [1], named dual Poynting effect, we show for the first time that the supercontraction induced shortening of the fiber may lead to a coupled torsion also in the more realistic case of structural molecules undergoing shortening oriented along the fiber direction. The effectiveness of the model is demonstrated also in quantitatively predicting the experimental torsion behavior of spider silks under increasing humidity. We argue that this result can be interesting also in the perspective of designing new simple humidity controlled torsional actuators.

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Fractional advection-diffusion equation for water monitoring systems

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Recent literature is rich in models based on fractional differential equations that can describe nonlinear phenomena in the fields of engineering, physics, and natural sciences. This has raised the interest of researchers in trying, if possible, to determine analytical solutions or, alternatively, approximated solutions gained by using suitable numerical methods [1, 2].

The main idea proposed in this work is to study an anomalous advection-diffusion process for water distribution systems [3]. The dynamics of water within distribution systems is characterized by different velocity regimes, ranging from laminar, transitional and turbulent, which lead to an advective and diffusive-dispersive behavior of the contaminant. This phenomenon, usually modeled by a classical advection-dispersion-reaction equation, through the introduction of a fractional derivative operator, will be modeled by a new fractional model, of advection-dispersion-reaction equation, which allows to describe anomalous processes.

The fractional model has been studied numerically, attempting the fitting real phenomena.

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Pattern formation in a Lotka–Volterra competition model with diffusion and advection

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In this talk, we will discuss the pattern formation properties of the following Lotka–Volterra competition model with diffusion and advection:

$$\begin{cases} u_t = \nabla \cdot (c_1 \nabla u + d_u u \nabla v) + \Gamma u \left(\mu_1 - \gamma_{11} u - \gamma_{12} v\right) & \text{in } \Omega \times (0,T), \\ v_t = \nabla \cdot (c_2 \nabla v + d_v v \nabla u) + \Gamma v \left(\mu_2 - \gamma_{21} u - \gamma_{22} v\right) & \text{in } \Omega \times (0,T), \\ \partial_\nu u = \partial_\nu v = 0, & \text{on } \partial\Omega \times (0,T), \\ u(\cdot,0) = u_0 \geq 0, \qquad v(\cdot,0) = v_0 \geq 0 & \text{in } \Omega, \end{cases} \tag{3}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, $\nu(x)$ is the outer unit normal vector at $x \in \partial\Omega$ and $\partial_\nu u = \nu(x) \cdot \nabla u$ is the out-flux of u.
The unknown functions $u(x,t)$ and $v(x,t)$ represent the population densities of of two competing species in the same habitat Ω. In the reaction terms of the Lotka–Volterra competition type, μ_1 and μ_2 are positive constants corresponding to the birth rates of the respective species, while γ_{11}, γ_{12}, γ_{21}, and γ_{22} are positive constants related to the inter-specific and the intraspecific competition. The parameter Γ gives the relative strength of reaction terms (or, alternatively, the size of the spatial domain and the time scale). The positive constants c_1 and c_2 portray the diffusion rates of the respective species. In the advection terms, d_u and d_v are non-negative constants, depicting the ecological tendency of each species to diffuse towards the low density areas of the other.
Notice that if both the advection terms $\nabla \cdot (u \nabla v)$ and $\nabla \cdot (v \nabla u)$ are replaced by the cross-diffusion term $\Delta(uv)$, one obtains the famous Shigesada-Teramoto-Kawasaki model [1].
We will show how the advection terms in Eq. (3) permit to model spatial segregation phenomena and are indeed responsible for the pattern formation. In a one-dimensional domain, near marginal stability, through a weakly nonlinear analysis, we manage to predict the shape and the amplitude of the pattern, for which we derive the cubic (in the supercritical case) and the quintic (in the subcritical case) Stuart–Landau equation.
In the case of a two-dimensional domain, we observe that the bifurcation can be regular, degenerate non-resonant and resonant, leading to different kind of solutions.

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Stationary patterns in a FitzHugh-Nagumo model with anomalous diffusion

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Anomalous diffusion phenomena frequently occur in natural systems. Interesting examples include autocatalytic chemical reactions on porous media, the preferential movement of species driven by safety or hunting strategies, and long-range interactions in ion channels within the plasma membrane. Cross-diffusion is a kind of nonlinear diffusion used to describe population dynamics where the gradient of one species induces the flux of the other species [1, 2]. On the other hand, super-diffusive processes, such as Lévy flights, can describe the mass diffusion in plasmas or foraging dynamics of birds and oceanic predators for randomly located resources and lead to fractional derivative modeling [3, 4].

In this talk, we investigate how anomalous diffusion influences the formation of stationary patterns in the FitzHugh-Nagumo model, which represents the paradigm system to describe excitable dynamics both in chemical reactions and population dynamics [5, 6]. We find that introducing anomalous diffusion terms allows for pattern formation in both short-range activation/long-range inhibition or long-range activation/short-range inhibition, relaxing the typical requirement for a rapidly diffusing inhibitor in the case of classical diffusion [7]. Moreover, in the presence of cross-diffusion, the spatial structures induced by long-range activation/short-range inhibition mechanisms are always out of phase (cross-Turing patterns) and subcritical in most of the instability regions [8].

Finally, using the formalism of amplitude equations, we derive the asymptotic profiles of the stationary solutions and classify the bifurcation, distinguishing between super- and sub-critical transitions.

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A brief history of maximum entropy principle in kinetic theory and non-equilibrium thermodynamics

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We present a brief history of how the well-known Maximum Entropy Principle has been used to close the moment equations of the Boltzmann equation. In particular, we highlight the important contributions of Kogan [1], as well as two foundational papers by Dreyer—one within the classical framework [2], and the other in a relativistic context [3]—which apply this principle and compare the results with the macroscopic theory of Rational Extended Thermodynamics [7, 8].

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Modeling and analysis of light-driven granule formation

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A model simulating the initial formation and growth of a phototrophic granular biofilm around wireless light emitter is proposed. The work focuses on the well-posedness and quantitative analysis of the model describing the initiation of the biofilm growth induced by cell attachment in the case of spherical geometry. Biofilm granule generates from the agglomeration of free living cells which lead to the formation of spherical-shaped microbial consortia. Under the hypothesis of radial symmetry, the granular biofilm is modeled as a free boundary domain, where the free boundary is the radius of the granule. The attached species proliferate by consuming dissolved substrates diffusively transported from the bulk liquid within the granule. Granule size evolves over time due to attachment flux, microbial growth and invasion. The evolution of the state variables, that is the concentrations of the microbial cells both in sessile and suspended form and the dissolved substrates, is governed by a nonlinear system of hyperbolic-elliptic differential equations, while the free boundary evolution is governed by an ordinary differential equation. Light intensity provided by the emitter is accounted in the model as a further state variable, as it affects the attachment velocity and metabolic activity of light-dependent microorganisms. By using the method of characteristics, the equations are converted into an equivalent integral system. Existence and uniqueness of solutions are discussed and proved for the attachment regime using fixed point strategies. A numerical study is performed to explore and investigate the role of light-dependent species on the granulation process and how light conditions and concentration of nutrients affect the attachment and granule initiation.

Dispersive behaviour of quasilinear hyperbolic waves on periodic background

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Waves propagating in media with periodic structures have attracted a lot of attention in recent years (see, for example, [1]). These media show interesting macroscopic properties, which may be quite different from those of the individual materials constituting the stratified system. More recently, waves on fluids and gases with an underlying periodic structure have been studied.

These waves present a peculiar, somehow unexpected, behaviour. For example, there is evidence that in spite of the fact that the waves are governed by genuinely quasi-linear hyperbolic system, they do not break into a shock [2, 3].

Here we present several systems in which hyperbolic waves show a dispersive behaviour, and deduce approximate effective equations which explain the peculiar phenomena.

In all cases, the effective equations are obtained by asymptotic expansion of the solution in a small parameter representing the ratio between the period of the structure and a typical wavelength.

We start considering 1D shallow water system with periodic bathymetry [2]. The detailed numerical solution of the system is compared with effective equations at various orders in the small parameter. Traveling wave solutions of the dispersive models are also computed and compared with the traveling waves emerging from the shallow water system.

As a second example we consider 2D shallow water [2], with waves propagating in the x-direction, while the bathymetry is periodic in the y direction.

The last example concerns the Euler equations in gas dynamics [4]. The stationary background is a state with constant pressure, zero velocity, and a periodic variation in the density.

Open problems and related work in progress will be mentioned.

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The KP equation in solid mechanics

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The propagation of nonlinear and dispersive waves in various physical contexts can be described by the well-known Kadomtsev–Petviashvili (KP) equation, a (2+1)-dimensional partial differential equation. In this paper, we demonstrate that the KP equation can be used to model the in-plane motion of compressible elastic solids with dispersion. Moreover, a modified KP equation featuring cubic nonlinearity is derived for the case of incompressible solids with dispersion. To the best of our knowledge, this represents the first rigorous and complete derivation of the KP equation within the framework of solid mechanics.

Vortex layers with small viscosity

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In this talk we shall consider the 2D Navier-Stokes equations

$$\begin{cases} \partial_t \omega + \mathbf{u} \cdot \nabla \omega = \nu \Delta \omega \\ u = \nabla^\perp \Delta^{-1} \omega \\ \omega(x, t = 0) = \omega_0(x) \end{cases}$$

with initial data concentrated on a small set and satisfying the bound

$$\int |\omega_0| dx \leq c.$$

A significant instance of this configuration is when the vorticity $\omega = O(\epsilon^{-1})$ is distributed close to a curve $y = \phi_0(x)$ and decays away from it on a scale $O(\epsilon)$, being $\epsilon > 0$ a small parameter. In the case of ideal flows, i.e., $\nu = 0$, an analysis [2] lead to the existence of solutions for a time independent from ϵ. For analytic data and for a short time, the solutions do not develop oscillations or concentrations. Moreover, the vorticity remains supported close to a curve $y = \phi(x, t)$ whose dynamics is ruled, in the limit $\epsilon \to 0$, by the Birkhoff-Rott equation. No hypothesis on the sign of the vorticity was imposed.

The aim of this talk is to discuss the viscosity effects, i.e., $\nu > 0$, assuming the diffusive scaling $\epsilon = O(\sqrt{\nu})$. We shall see that, for an almost flat layer, the vorticity follows, to the leading order, the Euler dynamics; and that the viscosity effects, confined to the correction equation, can be described by a controllable weakly nonlinear convection-diffusion equation.

We shall present some numerical work [1] highlighting the role of the Birkhoff-Rott singularity in the roll-up process of the vortex layer.

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Large time behavior of the solutions to the viscous MHD equations in $\mathbb{R}^2$

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We consider the large time behaviour of the solutions to the viscous magneto–hydrodynamic (MHD) equations in the whole space $\mathbb{R}^2$. The two dimensional MHD equations describe the evolution of an incompressible electrically conducting fluid moving through a magnetic vector field. The interaction between the fluid velocity and the magnetic field is described by the coupling between the Navier-Stokes equations and the Maxwell's equations. We consider vortex-current patchs as initial data.

Joint work with N. De Nitti, I. Kukavica, M.E. Schonbek and M. Sammartino.

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A two-dimensional wildfire propagation model with realistic topography and fuel mapping: numerical implementation and preliminary validation

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This study presents a two-dimensional wildland fire propagation model that integrates realistic wind fields and topographic data from a specific area in the municipality of Enna, Sicily. The model builds upon previous formulations [1, 2], describing fire dynamics through a system of reaction-diffusion equations:

$$\begin{cases} \rho C \dfrac{\partial T}{\partial t} + \rho C \mathbf{v} \cdot \nabla T - \nabla \cdot (K(T) \nabla T) + h(T)(T - T_a) = \rho Q \lambda Y \\ \dfrac{\partial Y}{\partial t} = -\lambda Y \end{cases}$$

where ρ is the air density, C is specific heat, $T(x,t)$ is temperature, $Y(x,t)$ is the solid fuel concentration, $\mathbf{v} = \mathbf{v}_W + \alpha \nabla h_T$ represents the combined effect of wind velocity $\mathbf{v}$ and terrain-driven advection (with ∇h_T is the local topography gradient and α a scaling parameter). The turbulent diffusivity $K(T)$ depends on temperature, while $h(T) = \bar{h}|T - T_a|^{1/3}$ models convective heat transfer, with T_a as ambient temperature. The combustion heat source $\rho Q \lambda Y$ includes the reaction rate $\lambda = s(T)A(T)e^{-E/RT}$, where $s(T)$ is an activation function, $A(T) = \bar{A}T^{1/2}$ is the pre-exponential factor, E is the activation energy, and R is the universal gas constant. Numerically, we compare two approaches: a Runge-Kutta scheme with finite differences and the Finite Element Method (FEM). While FEM has been tested so far only for flat terrain and random fuel distributions—showing promising stability and accuracy—the finite difference method has been successfully applied to real-world scenarios, incorporating detailed topography and heterogeneous fuel maps. These fuel distributions are derived from satellite imagery using clustering techniques. The results demonstrate how firefront propagation is influenced by the interaction of wind, topography, and fuel distribution. The finite difference method proved effective for complex scenarios, while FEM - currently applied to simplified cases - shows potential for future development. This study contributes to identifying optimal numerical methods for wildfire prediction.

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Hamiltonian reductions and effective wave models

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A systematic reduction approach is used to describe asymptotic long wave fully nonlinear models of wave motion in the Hamiltonian setting.

We start by considering Benjamin's Hamiltonian structure for a heterogeneous incompressible 2D Euler fluid. By applying the Marsden-Ratiu reduction scheme as in [1], we derive a Hamiltonian structure for a two-layer dispersive model. Then, using a double-scaling limit, we recover the canonical formulation of the Serre Green Naghdi (SGN) equations.

Finally, a coupled pair of SGN equations is shown to be equivalent to the Myhata-Camassa-Choi [2] model for a two-layer dispersive fluid and, through a Dirac reduction on a constrained manifold, the previously found Hamiltonian structure for this model is recovered.

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Modeling tumor-immune dynamics with therapy and delayed response

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I will present a system of delay differential equations for tumor-immune system interactions. The model incorporates the effects of adoptive cellular immunotherapy and the delay between tumor cell recognition and the subsequent immune response. Next, I will discuss the global well-posedness of the system, its equilibrium states and their asymptotic stability, and how the delay influences the dynamics and stability. Finally, I will present numerical simulations of the delayed system, illustrating the impact of the delay and validating the theoretical predictions. The results will be discussed in light of the theoretical analysis and interpreted within the context of tumor progression.

Vegetation patterns driven by autotoxicity: a cross-diffusion model

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In population dynamics, cross-diffusion describes the influence of one species on the diffusion of another and, surprisingly, even though the reaction part does not present the activator–inhibitor structure, cross-diffusion terms are often the key ingredient for the appearance of spatial patterns [1]. Furthermore, from the modelling perspective, cross-diffusion terms naturally appear in the fast-reaction limit of a "microscopic" model (in terms of time scales) presenting only standard diffusion and fast-reaction terms, thus incorporating processes occurring on different time scales [2].

We exploit this technique to model the auto-toxicity effect in plant growth dynamics [3], i.e. negative plant–soil feedback due to the decomposed biomass of the plant on its own growth. The "macroscopic" model presents a cross-diffusion term that allows the formation of spatial patterns without introducing water as a variable [4, 5]. A deeper understanding of the conditions required for non-homogeneous steady states to exist is provided by combining a detailed linear analysis with advanced numerical bifurcation methods via the continuation software `pde2path` and numerical simulations. Among the stable patterns observed, it is worth noting the presence of double-peak pulses of biomass in one-dimensional domains, which are equivalent to reinforced boundaries for the irregular spots forming on two-dimensional domains, for a wide region of parameter space.

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Shock structure and the sub-shock formation in a binary mixture of monatomic gases

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In this talk, we present the shock structure for an inert binary mixture of monoatomic gases [1] based on the multi–velocity and multi–temperature Grad's 13–moment equations [2]. By comparing the theoretical predictions with the experimental data on the velocity and temperature profiles in a mixture of He and Ar gases, we quantitatively show the present approach's usefulness.

As an extension of preliminary results [3], a complete classification of the sub-shock formation is done using the strategy developed in [4]. Namely, for the mass ratio values of all possible pairs of monatomic species, we classify the regions in the concentration–Mach number plane for which the sub-shock (discontinuous part) in the shock structure of one or both constituents may exist or not. We numerically analyze the shock–structure solutions for all classified cases, and confirm that at most two sub–shocks arise, one in each constituent.

We also discuss the loss of hyperbolicity of the system of field equations to clarify the validity range of the present approach, especially for high Mach numbers.

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Modeling the comparative influence of conjugation and transformation on plasmid spread in microbial communities

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Antimicrobial resistance (AMR) poses a critical global challenge with significant implications for public health and environmental sustainability. Antibiotic resistance genes (ARGs) are often carried by plasmids, mobile genetic elements which can transfer between microbial cells through horizontal gene transfer (HGT) mechanisms, primarily conjugation and natural transformation. These processes, along with the presence of metal resistance genes (MRGs) on plasmids, are amplified by metal contamination, which increases selective pressure and facilitates the spread of ARGs. This study presents a multidimensional continuum model for plasmid dissemination in microbial communities via horizontal gene transfer. The model is formulated as a system of nonlocal partial differential equations derived from mass conservation laws and reaction kinetics principles. The microbial domain is modeled as a homogeneous, viscous, incompressible fluid with a velocity given by Darcy's law. The model considers dynamics of plasmid-carrying cells and their horizontal gene transfer via conjugation and natural transformation. A convolution integral regulates the gene transfer expression to account for its dependence on the presence of potential receptors around a donor, called recipient-sensing. Numerical simulations explore the dynamics of plasmid spread in a two-dimensional system, investigating how transformation and conjugation affect microbial ecology and how metal dynamics regulate, and are regulated by, the community. Results align with experimentally observed trends in HGT intensity and the role of selective pressure in plasmid spread.

Kinetic methods for consensus-based segmentation

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Image Segmentation is a fundamental task in the context of image processing and computer vision that consists of partitioning an image into subsets of pixels that share similar properties so as to facilitate the analysis and interpretation of the visual data. There are a wide range of applications for this technique, particularly for the analysis of biomedical images.

In this talk I will present a new approach based on Consensus-Based Models for the Image Segmentation task [1, 3]. By considering each pixel as a particle characterize by a 2D vector position and a static feature we propose a virtual interaction scheme based on the Hegselman-Krause Model that will determine the asymptotic formation of a finite number of clusters [2]. I will discuss the application of this method to a variety of biomedical images. This work has been done with the collaboration of *Fondazione Mondino*.

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On the Holway-Weiss debate: convergence of the Grad-moment-expansion in kinetic gas theory

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Moment expansions are used as a model reduction technique in kinetic gas theory to approximate the Boltzmann equation [1]. Rarefied gas models based on so-called moment equations have become increasingly popular [2]. In particular, the regularized-13-moment equations (R13) [3] offers an attractive balance between physical accuracy - extending the validity of classical fluid dynamics, and complexity by restricting itself to a relatively low number of moments. Ultimately, it is possible to even show well-posedness of the R13 system in the linear case [4].

The way forward is to consider the nonlinear case and include more moments to achieve ever higher physical accuracy or consistency with the Boltzmann equation. However, in a seminal paper by Holway (1965) [5], a fundamental restriction on the existence of the expansion of the distribution function in terms of moments, questioning the validity of the entire approach moment approximation. Holway used this insight to explain subshock behavior of shock profile solutions obtained by moment equations. Later, Weiss (1996) [6] found continuous or non-sub-shock solution for shock profiles and studied Holway's argument in detail concluding that there is an error. Weiss argued that a restriction on moment equations does not exist.

We will revisit and discuss Holway's and Weiss' findings and explain that both arguments have a correct and an incorrect part [7]. While a general convergence restriction for moment expansions does exist, it cannot be attributed to subshock solutions. We will also discuss the implications of the restriction and give some numerical evidence for our considerations.

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Urban evolution: a dynamic model for a stable and resilient development

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Since the middle of the XX century the world's population has seen a sharp increase due to a general improvement in living conditions. Over the past 35 years the urban population has jumped by 2.3 billion people and analyses indicate a consolidated trend, showing that, by 2050, 68% of the global population will live in cities. Managing urban processes and determining which tools can be used to achieve this result has, therefore, become a central issue for governing such expanding and complex systems, with relevant environmental consequences. Suitable policies in planning the future development of cities are a need to which scholars, planners and public managers are struggling to answer, facing an urban development unthinkable just 30 years ago. The 11th SDG ("Make cities and human settlements inclusive, safe, resilient and sustainable") proposes several targets to reach and actions to perform in a variety of fields. The concept of sustainability has progressively expanded incorporating not only efficiency and resilience, but also, even more critically, social aspects. It is the so-called human factor, which must be fully considered for cities' sustainable development.

Building on this premise, it is clear how the behaviour of any urban system should be considered as the result of a continuous balance between whatever urban variable and the human-induced effects on that variable, leaving behind any top-down management solution. One way to conceptualize such a situation is through mathematical models, joining together the human factor and one or more specific urban elements related to sustainability, as already done for natural landscape territories [1, 2]. The opportunity of guiding urban development towards preferred directions can't be neglected, but the tool of a dynamical model can be of assistance showing the eventual changes to be applied to reach that target. Such a model is the focus of our research. The initial approach examines two state variables, considering pedestrianisation and people's opinion, and deliberately excludes any indicator of urban growth [3]. A detailed study of its dynamics and stability was conducted [3].

Subsequently, a third state variable, of an economic nature, was introduced, leading to the final three-variable model, and the impact of the additional human factor on the dynamics previously defined was analysed. For both models, equilibrium states and their asymptotic stability have been studied, and the effects of Hopf bifurcation and the behaviour beyond have been explored. The results have been interpreted in terms of the urban meaning, aiming to bridge the gap between mathematical modelling and urban planning.

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How to measure multidimensional variation?

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The coefficient of variation, which measures the variability of a distribution from its mean, is not uniquely defined in the multidimensional case, and so is the multidimensional Gini index, which measures the inequality of a distribution in terms of the mean differences among its observations[2]. In this talk, we aim to connect these two notions of sparsity, and propose a multidimensional coefficient of variation based on a multidimensional Gini index. We demonstrate that the proposed coefficient possesses the properties of the univariate coefficient of variation. We also show its connection with the Voinov-Nikulin coefficient of variation, and compare it with the other multivariate coefficients available in the literature. Examples will clarify the applicability of the new notion to measure inequalities in a multidimensional setting.

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The extended thermodynamics for the fractional exclusion statistics

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We consider the non-equilibrium theory for the fractional exclusion statistics (FES) by using the Maximum Entropy Principle and the Extended Thermodynamics (ET) [1, 2]. Many thermodynamic properties of an ideal gas that obeys the FES have been studied in literature under equilibrium conditions [3, 4, 5, 6] and in particular the expressions for the distribution function have been determined using the Maximum Entropy Principle (MEP) both in the framework of a semiclassical local theory [3] than of a non-local quantum Wigner theory (Quantum MEP) [7]. The aim of this talk is to develop and apply a general theory for the fractional exclusion statistics in *non-equilibrium conditions* by connecting the theory, in a semiclassical framework, with the Boltzmann transport equation (BTE) and with the method of moments in ET. In particular: (i) The entropy balance equation is determined and the statistical consequence of theory are discussed. (ii) Both the entropy and its flux are computed explicitly in terms of the non-equilibrium Lagrange multipliers while, by using a general expression for the energy dispersion relation, some thermodynamic properties are explicitly analyzed. (iii) The construction of an arbitrary set of closed *hydrodynamic equations* (HD), in the context of Extended Thermodynamics, is explicitly obtained. (iv)The hyperbolicity of HD evolution equations, in the case of system called *ultrarelativistic* [6], is analyzed.

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Viscoplastic flows in channels with small aspect ratio: 'exact' versus regularised models

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Several materials like mineral slurries, paints, suspensions, crude oils, and extrusions in 3D printing are modelled as viscoplastic fluids. The main constitutive feature of these models is represented by the so-called *yield stress*, namely a threshold to be exceeded for the fluid to start flowing. Based on the pioneering studies by [1] and [2], several constitutive models for the mechanical response of viscoplastic fluids have been proposed. All these models are characterized by the presence of a parameter that accounts for the critical threshold for the shear stress.

It must be said that still nowadays the existence of a real yield stress is the subject of an open debate. Indeed, the existence of a threshold for the stress above which the fluid starts flowing might be just a consequence of the experimental techniques and apparatuses used to investigate the rheology of the materials. Whether or not a fluid exhibits a yield stress might be due to the time and length scales of the observations and of the applied forces.

On the other hand, the numerical integration of the equations governing the flows of fluids that seem to exhibit a yield stress is very challenging. When the shear stress attains the critical limit, the effective viscosity diverges and thus no straightforward computation is possible. To overcome this difficulty, regularisation methods have been proposed. They consist in replacing the singular constitutive model with a smooth approximation that depends on a suitable positive *regularisation parameter* (say ϵ). In the limit as $\epsilon \to 0^+$, the regularised model tends pointwise to the exact viscoplastic model.

Introducing a regularised model is natural from a mathematical point of view as smoothing a non-differentiable minimisation problem allows the use of advantageous variational techniques that can be implemented in highly performing numerical schemes. The introduction of a regularised model is justified also from a physical point of view as the regularised effective viscosity approaches a large but *finite* value as the shear rate tends to zero. Since no one is able to establish with absolute certainty whether or not a fluid actually exhibits a yield stress, a regularised model may be more physically plausible than an exact viscoplastic one.

Particular attention has to be paid to the convergence of the flow predicted by the regularised model to the one predicted by the 'exact' model as $\epsilon \to 0^+$. Limiting to Bingham fluids, the convergence in mean of the regularised flows to the exact flows has been addressed in very few studies. In particular, for steady Stokes flows in a bounded two-dimensional domain with Lipschitz boundary subjected to Dirichlet boundary conditions, the difference between the exact Bingham and regularised flows is estimated in the H^1-norm as ϵ^α, with $\alpha \in (0, 1)$ depending essentially on the particular regularised model used. However, the location of the yield surface (which is undoubtedly the main problem in viscoplastic flows) lies outside the convergence in mean of the regularised velocity field, since it is linked to the convergence of the extra stress. To show that the regularised yield surface converges to the one predicted by the exact non-smooth model one needs results on the uniform convergence of the regularised stress to the exact stress which, except for very few simple cases (e.g. flow in a flat channel), has not yet been proven.

The main purpose of this talk is to discuss to what extent the flows predicted by the 'exact' Bingham or Herschel-Bulkley models and their regularisations differ. In particular, we highlight the possible differences between the regularised and 'exact' yield surfaces in a viscoplastic fluid flowing in a narrow symmetric channel. Moreover, we show that, irrespective of the regularisation considered, in the limit as $\epsilon \to 0^+$, the regularised flow and yield surface tend to expressions that depend exclusively on the profile of the channel walls. The resulting asymptotic yield surface coincides with the yield surface predicted by the 'exact' Bingham or Herschel-Bulkley models when the fluid flows in a plane channel. If the flow takes place in a non-plane channel, the two surfaces are markedly different. This means that only in plane channels regularised models envisage, in the limit as $\epsilon \to 0^+$, a rigid core like when considering the 'exact' Bingham or Herschel-Bulkley models.

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Self-similar solutions to the radially symmetric relativistic Euler system with Synge energy

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We study the initial-boundary problem of the relativistic Euler system for monatomic gases and diatomic gases with Synge energy. For the relativistic Euler equations with radial symmetry, based on detailed analysis of the corresponding nonlinear ordinary differential equations and some new estimates of the Bessel functions, global entropy solutions are constructed. This is a joint work with Tommaso Ruggeri and Ferdinand Thein.

Finite time blow-up for consensus dynamics and applications

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We introduce a new class of kinetic equations to explore mass-dependent effects in Fokker-Planck-type models originally developed for quantum indistinguishable particles in a spatially homogeneous setting. By analyzing the resulting PDE governing particle density evolution, we characterize the regimes in which a critical mass leads to finite-time blow-up of the solution. Finally, we discuss the implications of these findings for global optimization problems.

List of speakers in alphabetical order

Alì Giuseppe, Università della Calabria
Amabili Marco, Westlake University
Amendola Ada, Università degli Studi di Salerno
Aoki Kazuo, National Cheng Kung University
Arima Takashi, National Institute of Technology, Tomakomai College
Arnone Giuseppe, Istituto Nazionale di Alta Matematica
Averga Sabrina, Università di Pavia e Università degli Studi di Napoli Federico II
Bernhoff Niclas, Karlstad University
Bigoni Davide, Università di Trento
Binetti Claudia, Politecnico di Bari
Borsoni Thomas, CERMICS, ENPC, Champs-sur-Marne
Borsotti Jacopo, Università di Parma
Brini Francesca, Università di Bologna
Brocchieri Elisabetta, University of Graz
Buonomo Bruno, Università degli Studi di Napoli Federico II
Caflisch Russel, Courant Institute, New York University
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Celletti Alessandra, Università degli Studi di Roma Tor Vergata
Colombaro Ivano, Libera Università di Bolzano
Conte Martina, Politecnico di Torino
De Castro Motta Julia, Università di Salerno
De Luca Roberta, Università degli Studi di Napoli Federico II
Demarco Francesco, Università della Calabria
Desvillettes Laurent, Université Paris Cité
Di Feola Angelica Pia, Università della Campania "Luigi Vanvitelli"
Di Stefano Salvatore, Università degli Studi di Bari Aldo Moro
Dumbser Michael, Università di Trento
Falsaperla Paolo, Università di Catania
Fazio Claudia, Università degli Studi di Messina
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Goriely Alain, University of Oxford
Gouin Henri, Aix-Marseille University

Grifò Gabriele, INDAM – Istituto Nazionale di Alta Matematica "Francesco Severi"
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Iuorio Annalisa, Università degli Studi di Napoli Parthenope
Lacitignola Deborah, Università degli Studi di Cassino e del Lazio Meridionale
Liu Tai Ping, Taiwan & Stanford University
Lombardo Maria Carmela, Università degli Studi di Palermo
Loy Nadia, Politecnico di Torino
Lucchin Enrico, Università di Parma
Macaluso Anna, Università di Parma
Magni Francesco, SISSA
Mainardi Francesco, Università di Bologna
Manganaro Natale, Università degli Studi di Messina
Mattei Maria Rosaria, Università degli Studi di Napoli Federico II
Menale Marco, Università degli Studi di Napoli Federico II
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