IMPLEMENTING FLUTTER INSTABILITY IN A METAMATERIAL

Davide Bigoni[1*]
[1]*Instabilities Lab, University of Trento, Trento, Italy*

Summary Rigorous homogenization theory is applied to periodic elastic structures subject to prestress, with the purpose of obtaining equivalent elastic solids, where material instability may occur in a controlled way. The mentioned instabilities may be in the form of shear bands or more complicated Hopf bifurcations. Within the context of shear band formation, this is shown to be possible for both compressive and tensile stress states. Hopf bifurcation is obtained by harnessing flutter instability of elastic structures. A way to the achievement of the latter instability is pursued through non-holonomic constraints. These constraints allow to maintain conservation of the energy and open the way to odd elasticity.

INTRODUCTION

A design strategy leading to metamaterials capable of effectively filtering and conditioning wave propagation is the composition of elastic structures via periodic lattices. In these structures, different effects related to out-of-plane or in-plane deformations, presence of bending moment or prestress, have been so far explored. We investigate the possibility of defining structured materials capable of suffering instabilities for different prestress states, including tensile. In particular, Floquet-Bloch wave asymptotics is used to homogenize the in-plane mechanical response of elastic periodic grids, so to obtain an equivalent prestressed elastic solid subject to incremental time-harmonic vibration, which includes, as a particular case, the incremental quasi-static response. The equivalent elastic solid is obtained from its acoustic tensor, directly derived from homogenization. Shear band formation in the equivalent continuum coincides with macro-bifurcation in the lattice, while micro-bifurcation remains undetected in the continuum and corresponds to a vibration of vanishing frequency of the lowest dispersion branch of the lattice, occurring at finite wavelength [1]. Using this approach, architected materials can be designed with tuned instabilities. Moreover, introducing sliders in the elastic grid it is shown that a bounded stability domain can be achieved for the equivalent material [2].

FLUTTER INSTABILITY AND HOPF BIFURCATION IN SOLIDS

The Floquet-Bloch wave asymptotics developed in [1] is used to include in the homogenization scheme follower forces, see Figure 1. These are known to lead to flutter instability in structures [3]. The homogenization scheme yields an effective material, given through its unsymmetric acoustic tensor. This unsymmetry proves that the effective elastic material does not admit a strain potential and thus is a hypo-elastic or a Cauchy-elastic material [4]. The effective hypo-elastic material is found to suffer instabilities, in particular, flutter instability (Figure 2), a Hopf bifurcation which was advocated, although never indisputably experimentally found, to be possible in plastic materials, but not in elastic ones. The route shown here opens the way to the realization of elastic materials violating hyperelasticity. The follower forces, necessary for the instability, can be generated as the interaction with an 'environment', from which the material can absorb energy, so that production or release of work in a closed strain cycle does not violate thermodynamics.

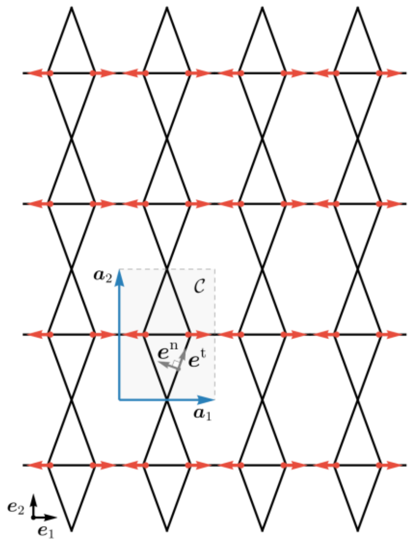

Figure 1. The elastic grid prestressed with follower forces and showing flutter instability.

* Corresponding author. Email: davide.bigoni@unitn.it

The realzation of the follower forces may be obtained though nonholonomic constraints [5] or smooth constraints with discontinuous curvature [6].

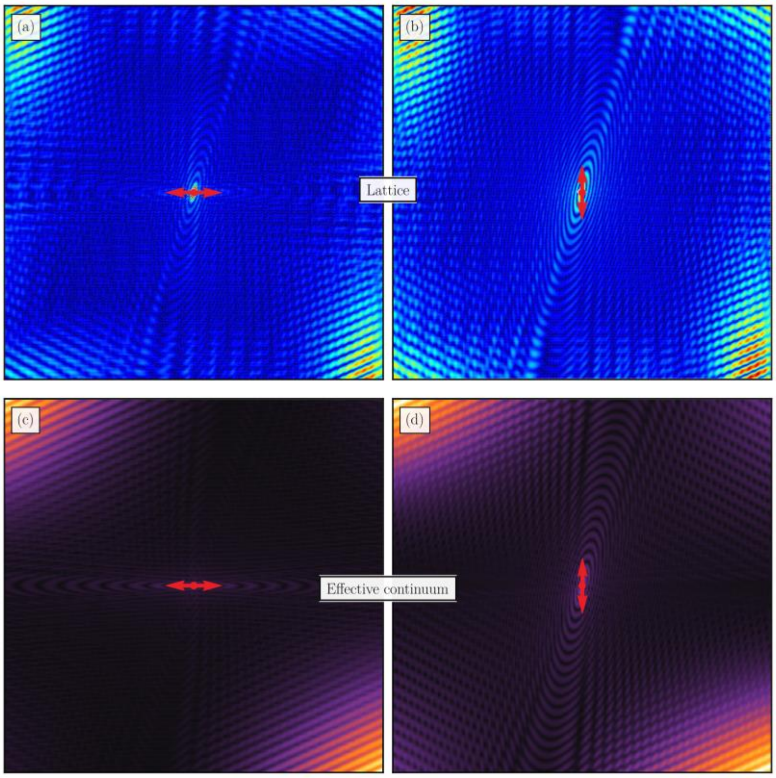

Figure 2. Time-harmonic forced response of the anisotropic lattice (upper part) and of the corresponding effective continuum (lower part), displaying flutter instability. The response is typical of the behavior of a dynamically unstable material, evidencing blowing-up waves traveling away from the forcing source but localized along two preferential directions. The latter may be captured by the spectral analysis of the effective acoustic tensor, characterized by complex-conjugate eigenvalues.

CONCLUSIONS: BEYOND HYPERELASTICITY

A rigorous application of homogenization theory shows that passive elastic materials without a strain energy potential are possible. The realization of these materials opens a new territory extending beyond hyperelasticity.

ACKNOWLEDGEMENTS

Financial support from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant agreement No. ERC-ADG-2021-101052956-BEYOND) is gratefully acknowledged.

References

[1] Bordiga G. Cabras L. Piccolroaz A. Bigoni, D. Dynamics of prestressed elastic lattices: Homogenization, instabilities, and strain localization. *J. Mech. Phys. Solids* **146**, 104198, 2021.

[2] Bordiga G. Bigoni D. Piccolroaz A. Tensile material instabilities in elastic beam lattices lead to a closed stability domain. *Phil. Trans. Royal Soc. A* **380**, 20210388, 2022.

[3] Bigoni D. Kirillov O. Misseroni D. Noselli G. Tommasini M. Flutter and divergence instability in the Pfluger column: Experimental evidence of the Ziegler destabilization paradox. *J. Mech. Phys. Solids* **116**, 99-116, 2018.

[4] Bordiga G. Piccolroaz A. Bigoni D. A way to hypo-elastic artificial materials without a strain potential and displaying flutter instability. *J. Mech. Phys. Solids* **158**, 104665, 2022.

[5] Cazzolli A. Dal Corso F. Bigoni D. Non-holonomic constraints inducing flutter instability in structures under conservative loadings. *J. Mech. Phys. Solids* **138**, 103919, 2020.

[6] Rossi M. Piccolroaz A. Bigoni, D. Fusion of two stable elastic structures resulting in an unstable system. *J. Mech. Phys. Solids* **173**, 105201, 2023.